Math 511

Q's

1.1 #7

$2x_1 + x_2 = 3$
$4x_1 + 3x_2 = 5$ $\rightarrow$ $\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 4 & 3 & 5 \end{array}\right] \xrightarrow[\text{ops}]{\text{row}} \left[\begin{array}{c|c} & \\ \end{array}\right]$

triangular

$2x_1 + x_2 = -1$
$4x_1 + 3x_2 = 1$ $\rightarrow$ $\left[\begin{array}{cc|c} 2 & 1 & -1 \\ 4 & 3 & 1 \end{array}\right]$

use $\left[\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 4 & 3 & 5 & 1 \end{array}\right] \rightarrow \left[\begin{array}{cc|cc} 1 & 1/2 & 3/2 & -1/2 \\ 4 & 3 & 5 & 1 \end{array}\right]$

ones as lead variable

1.2 gauss elimination $\rightarrow$ row echelon form $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

zero below lead variable

gauss-jordan elimination $\rightarrow$ reduced row echelon

ones as lead

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

zeros above lead

zeros below

lead one

$\left[\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 4 & 3 & 5 & 1 \end{array}\right]$ $-2r_1 + r_2 = N r_2$ $\left[\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{array}\right]$

$x_2 = -1$

$2x_1 + 1x_2 = 3$

$2x_1 - 1 = 3$

$x_1 = 2$

$\boxed{(2, -1)}$

$x_2 = 3$

$2x_1 + 3 = -1$

$x_1 = -2$

$\boxed{(-2, 3)}$

Gauss-Jordan

$\rightarrow$ Gauss

$\left[\begin{array}{cc|cc} 2 & 1 & 3 & -1 \\ 0 & 1 & -1 & 3 \end{array}\right]$ $(-1)r_2 + r_1 = N r_1$ $\left[\begin{array}{cc|cc} 2 & 0 & 4 & -4 \\ 0 & 1 & -1 & 3 \end{array}\right]$

$\frac{1}{2}r_1 = Nr_1$

$$\left[\begin{array}{cc|cc} 1 & 0 & 2 & -2 \\ 0 & 1 & -1 & 3 \end{array}\right]$$

System 1 $(2, -1)$

System 2 $(-2, 3)$

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row-echelon

ex

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

ex

$y + 4 = 4$

$$\begin{aligned} 2y + 2u &= 8 \\ x - 2y + z + u &= 0 \\ 2x + y + z - u &= 3 \\ x + y + u &= 5 \end{aligned}$$

$$\left[\begin{array}{cccc|c} 0 & 2 & 0 & 2 & 8 \\ 1 & -2 & 1 & 1 & 0 \\ 2 & 1 & 1 & -1 & 3 \\ 1 & 1 & 0 & 1 & 5 \end{array}\right]$$

$\frac{1}{2}$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 1 & 4 \\ 1 & -2 & 1 & 1 & 0 \\ 2 & 1 & 1 & -1 & 3 \end{array}\right]$$

$-1r_1 + r_3 = Nr_3$

$-2r_1 + r_4 = Nr_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 1 & 4 \\ 0 & -3 & 1 & 0 & -5 \\ 0 & -1 & 1 & -3 & -7 \end{array}\right]$$

$3r_2 + r_3 = Nr_3$

$r_2 + r_4 = Nr_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & -2 & -3 \end{array}\right]$$

$r_4 - r_3 = Nr_4$

$r_3 - r_4 = Nr_4$

then $\frac{1}{5}r_4$

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 5 \\ 0 & 1 & 0 & 1 & 4 \\ 0 & 0 & 1 & 3 & 7 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

Gauss-Jordan:

$$\left[\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right] \qquad \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right]$$

$$\boxed{(x, y, z, u) = (1, 2, 1, 2)}$$

Solve by Gauss (+ Back Solve)
Solve by Gauss-Jordan

Types

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$m \times n$ Matrix

$(m = n)$ Determined System — possible solns
1. 1 - soln
2. ∞ - soln's
3. 0 - soln's

$(m > n)$ Overdetermined — possible solns
1. 1 - soln
2. ∞ - soln's
3. 0 - soln's

$(m < n)$ underdetermined — possible soln's
1. ∞ soln's
2. 0 - soln's

Gauss form of possible soln's

1. 1 - soln

$$\left[\begin{array}{ccccc|c} 1 & \cdot & \cdot & & & \\ 0 & 1 & \cdot & & & \\ 0 & 0 & 1 & \cdots & & \\ 0 & 0 & 0 & 1 & & \\ 0 & 0 & 0 & 0 & & 0 \\ 0 & 0 & 0 & 0 & & 0 \end{array}\right]$$

2. 0 - soln's

Special row

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & \cdots & 0 & 1 \end{array}\right]$$

$0x_1 + 0x_2 + \cdots + 0x_n = 1$

$0 = 1$

③ ∞ - sol'n's

x, y, z

ex

$$\begin{array}{c} \text{lead} \quad \quad \text{lead} \quad \text{lead} \\ \begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \end{array} \\ \left[\begin{array}{cccc|c} 1 & 2 & 3 & 4 & 1 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \\ \text{free (under } x_2\text{)} \end{array}$$

x_2 = any # at all

$\alpha \in \mathbb{R}$

let $x_2 = \alpha$

1) $x_4 = 2$

$x_3 = -4$

$x_2 = \alpha$

$x_1 + 2\alpha + (-12) + 8 = 1$

$x_1 = 5 - 2\alpha$

$$\boxed{(5-2\alpha, \alpha, -4, 2)}$$