Math 511

Q's 6b

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 2 & -2 & 1 & 2 & 8 \\ 3 & 1 & 2 & -1 & -1 \end{array}\right] \end{array} \xrightarrow[-3r_1+r_3\to]{-2r_1+r_2\to} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & 8 & 1 & 4 & 10 \end{array}\right] \xrightarrow{r_2+r_3} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 1 & 3 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right]$$

3x4

under determined → ∞ soln? or → 0 soln?

$$r_1 - r_3 \quad \left[\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 0 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \quad 8r_1 + 3r_2 = Nr_1 \quad \begin{array}{c} \left[\begin{array}{cccc|c} 8 & 0 & 5 & 0 & 6 \\ 0 & -8 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \\ \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \end{array} \to \begin{array}{c} \left[\begin{array}{cccc|c} 1 & 0 & 5/8 & 0 & 6/8 \\ 0 & 1 & 1/8 & 0 & -1/4 \\ 0 & 0 & 0 & 1 & 3 \end{array}\right] \\ \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \end{array}$$

x_3 is free $\qquad x_3 = \alpha$

Homogeneous Systems of Linear Equations

$$\left.\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = 0 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = 0 \end{array}\right\}$$

has an obvious soln. (trivial soln)

$x_1 = 0, x_2 = 0, \ldots, x_n = 0$

has ∞ or 0 soln's

Thm: if $m < n$ (under det system) for homogeneous system then it has an infinite number of solns or we can say it has non-trivial solutions.

1.3 Matrices / Vectors

New toy: rect. block of numbers (Matrix)

ex $\begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ $\qquad A = \begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

Notation: $\underset{m\times n}{A} = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & & & \\ a_{m1} & - & - & a_{mn} \end{bmatrix}$

Special cases: $1 \times n$ or $m \times 1$ column vector

$[v_1\ v_2\ \cdots\ v_n]$ $\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_m \end{bmatrix}$

row vector

Notation: bold font lower case $\mathbf{v}$ col. vector

$\vec{\mathbf{v}}$ row vector

Arithmetic

#1 $A = B$ ① both are $m \times n$

② $a_{ij} = b_{ij}$ for all i, j

#2 $A + B$ ① both are $m \times n$

② $[a_{ij} + b_{ij}]$

#3 $\alpha A = [\alpha a_{ij}]$ Scalar multiplication

#4 Matrix $\cdot$ Matrix? Matrix multiplication $3 \times 2 \quad 2 \times 3$

$$\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 2 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ -1 & 1 \end{bmatrix}} = \underset{2\times 2}{\begin{bmatrix} -2 & 6 \\ 2 & 0 \end{bmatrix}}$$

scalar product $[1 \quad 2 \quad 3] \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = 1\cdot 1 + 2\cdot 0 + 3\cdot -1 = \underline{\underline{-2}}$

single real number

Scalar product: $\vec{x}\,y = x_1y_1 + x_2y_2 + \dots + x_ky_k$ = Scalar

row vector, col vector

why? $a_1x_1 + a_2x_2 = b$

$$\begin{bmatrix} a_1 & a_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

also $$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & \dots & a_n \end{bmatrix}$$

Next: ① $A x$ (matrix, col. vector) ② $A \cdot B$

$$\begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vec{a}_m \end{bmatrix} x \qquad \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{bmatrix} \begin{bmatrix} b_1 & b_2 & \dots & b_n \end{bmatrix}$$

$$\cancel{\begin{matrix} 3x_1 + 2x_2 \\ -1x_1 + 3x_2 \end{matrix}} \rightarrow Ax = \begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$