Math 511

~~(underdet)~~

homogeneous $\to$ 1 soln (only trivial)
$\to$ ∞ soln (free variable $\to \infty$ soln)
$\to$ ~~$\emptyset$ soln's~~ (inconsistent)

Q's a)

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 3 & 0 \\ -1 & 1 & \beta & 0 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 3 & \beta+1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & \boxed{\beta-2} & 0 \end{array}\right]$$

a) homogeneous systems always have at least trivial $x_1=0, x_2=0, x_3=0$ soln. They can never be inconsistent

b)

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & \beta-2 & 0 \end{array}\right]$$

x_1, not free

x_2, not free

x_3?

$\to$ $\beta-2=0$ or $\beta=2$
$\to$ x_3 is free and ∞ soln's

$\to$ $\beta-2\neq0$ or $\beta\neq2$
$\to$ x_3 is not free and unique soln is trivial soln.

6d)

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1/3 & 0 \end{array}\right]$$

Arithmetic

keys: A, $\vec{v}$, v

rules: $A = B$, $A + B$, αA

① Matrix-Vector (col)

$A v$

Scalar product $\vec{x} y$

$= x_1 y_1 + x_2 y_2 + \dots + x_n y_n$

a) $$A v = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{bmatrix} v = \begin{bmatrix} \vec{a}_1 v \\ \vec{a}_2 v \\ \vdots \\ \vec{a}_m v \end{bmatrix}$$

or

$$A v = [a_1 \; a_2 \dots a_n] \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix} = \underbrace{v_1 a_1 + v_2 a_2 + \dots + v_n a_n}_{\text{linear combo of A's columns}}$$

b) $A \cdot B$ (matrix multiplication)

$$= \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{bmatrix} [b_1 \; b_2 \dots b_n] = \begin{bmatrix} \vec{a}_1 b_1 & \vec{a}_1 b_2 & \dots & \vec{a}_1 b_n \\ \vdots & & & \\ \vec{a}_m b_1 & - & - & \vec{a}_m b_n \end{bmatrix}$$

so $$\underset{m\times k}{A} \; \underset{k\times n}{B} = \underset{m\times n}{\Big[\underset{1\times k}{\vec{a}_i} \; \underset{k\times 1}{b_j} \Big]}$$

ex

$$\underset{2\times 4}{\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}} \underset{4\times 3}{\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}} = \underset{2\times 3}{\begin{bmatrix} \square & \square & \square \\ \square & \boxed{0} & \square \end{bmatrix}}$$

Swap col. rows

A^T. If $A = [a_{ij}]$, then $A^T = [a_{ji}]$

$$\underset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T} = \underset{3\times 2}{\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}}$$

Thm (based on $A\mathbf{x} = \underbrace{x_1 a_1 + x_2 a_2 + \dots + x_n a_n}_{\text{linear combo of A's columns}}$)

Solve $A\mathbf{x} = \mathbf{b}$
has a soln (it is consistent) if there exists
a linear combination of A's cols. for $\mathbf{b}$.

$$x_1 a_1 + x_2 a_2 + \dots + x_n a_n = \mathbf{b}$$

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toys (+) rules → laws?

Matrix Algebra

laws for $A+B$

① $A + B = B + A$

② $(A+B)+C = A + (B+C)$

for AB

Note: Matrix mult. is not commutative.

③ $(AB)C = A(BC)$

Distrib

④ $A(B+C) = AB + AC$

⑤ $(B+C)A = BA + CA$

αA ⑥ $(\alpha\beta)A = \alpha(\beta A)$

⑦ $\alpha(AB) = (\alpha A)B = A(\alpha B)$

⑧ $(\alpha+\beta)A = \alpha A + \beta A$

⑨ $\alpha(A+B) = \alpha A + \alpha B$

A^T ⑩ $(A^T)^T = A$

⑪ $(\alpha A)^T = \alpha A^T$

⑫ $(A+B)^T = A^T + B^T$

⑬ $(AB)^T = B^T A^T$

Special Matrices:

Identities → Additive Identity $O = [0]_{n\times n}$

→ Matrix Mult. Identity $I = [\delta_{ij}]$

$n\times n$

$$= \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

Undo? - Inverse