Math 511

Q's

inconsistent = no soln.

$$\left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 4 & 1 \end{array}\right] \to \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 0 & -1 \end{array}\right]$$

$0 \neq -1$

no soln.

ex (2x2) thm: $A\mathbf{x} = \mathbf{b}$ has a soln iff.

$$x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 = \mathbf{b}$$

linear combo of $\mathbf{a}_1$, $\mathbf{a}_2$ that makes $\mathbf{b}$.

ex

$$\left[\begin{array}{cc|c} 1 & 2 & 4 \\ -1 & 3 & 1 \end{array}\right] \to \text{"see"} \quad 2\begin{bmatrix} 1 \\ -1 \end{bmatrix} + 1\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

consistent and soln is $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

vs Gauss jordan

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 1 \end{array}\right]$$

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$A = [a_{ij}]$ $\quad B = [b_{ij}]$ $\quad C = [c_{ij}]$

$$A + B = [a_{ij} + b_{ij}]$$

$$B + A = [b_{ij} + a_{ij}] = [a_{ij} + b_{ij}] = A + B$$

ex

$$d_{il} = \sum_{k=1}^{n} a_{ik} b_{kl} = a_{i1}b_{1l} + a_{i2}b_{2l} + a_{i3}b_{3l} + \dots + a_{in}b_{nl}$$

($\mathbf{b}_l$ brackets the b terms; $\vec{a}_i$ brackets the a terms)

$$AB = D$$

Identity of an operator

$A + B$ Identity is $\mathbb{O} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}$

Inverse of objects

$$A + (-1)A = \mathbb{O}$$
$$(-1)A + A = \mathbb{O}$$

$A \cdot B$ Identity $I = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix} = [\delta_{ij}]$

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$$

Def: A is invertable / non-singular if there is a matrix, B, such that

$AB = I$

$BA = I$ call B to be A's matrix multiplicative inverse

Notation: A^{-1}

Def: A is singular (concept: $A\underset{\text{non-zero}}{x} = \mathbb{O}$, $\exists$ non-zero)

A is non-invertable

Property A, B are non-singular then AB is non-singular and $(AB)^{-1} = B^{-1}A^{-1}$

1.5 Elementary Matrices

- When does A^{-1} exist?
- How to find A^{-1}?
- Can you factor A?

$A = L \cdot U$ ← upper triangular matrix $\begin{bmatrix} \diagdown & \\ 0 & \diagdown \end{bmatrix}$

L ↑ lower triangular matrix $\begin{bmatrix} \diagdown & 0 \\ & \diagdown \end{bmatrix}$

Systems of Eqn's → $\boxed{A\mathbf{x} = \mathbf{b}}$

↓ Solve (gauss, gauss-jordan)

row ops

$\mathbf{x} = \boxed{\text{ans.}}$

$A\mathbf{x} = \mathbf{b}$ vs M is invertible

consider $MA\mathbf{x} = M\mathbf{b}$

Soln is $\hat{\mathbf{x}}$

$MA\hat{\mathbf{x}} = M\mathbf{b}$

$\underbrace{M^{-1}M}_{I}A\hat{\mathbf{x}} = \underbrace{M^{-1}M}_{I}\mathbf{b}$

$A\hat{\mathbf{x}} = \mathbf{b}$

So We can mult. by non-singular matrices!

System of eqn's (vs) $A\mathbf{x} = \mathbf{b}$

row ops ← find matrices that do exactly row ops and are non-singular → $EA\mathbf{x} = E\mathbf{b}$

E ← "elementary" matrix

So Row Ops as Elementry Matrices

type 1 row swap $E_{type1} = I$ with row i,j swapped.

then $E_{type1}A = A$ with row i,j swapped

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 7 & 8 & 9 \\ 4 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$$

and $E_{type1}^{-1} = E_{type1}$

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type 2 λ row i = New row i

$E_{type2} = I$ with λ in i diagonal spot

Ex
$$\begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} -3 & -6 & -9 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

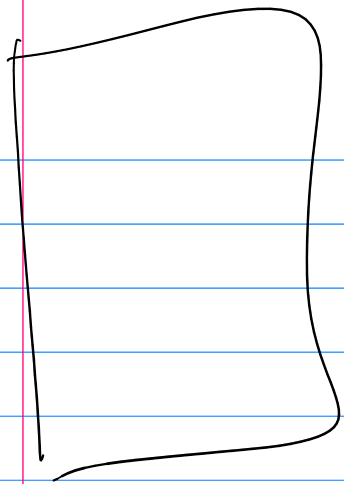
$E_{type2}^{-1} = I$ with $\frac{1}{\lambda}$ in i diagonal

Type 3 row i + m row j = New row i

$E_{type3} = I$, put m in i,j spot

$$\begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 7 & 8 & 9 \end{bmatrix}$$

$r_2 + (-4)r_1$ = New r_2



$$Ax = b$$

$$EAx = Eb$$

$$E_2 E_1 A x = E_2 E_1 b$$