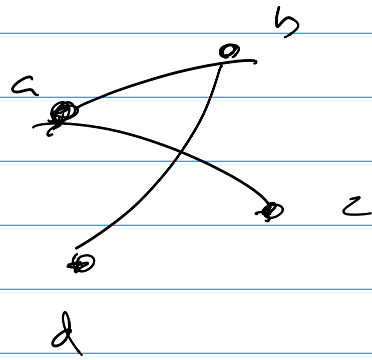
# Math 511

Q's

## Adj. Matrices




$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

## Elementary Matrices  E A

(row swap) $E_{type1}$ = $I$ with row $i, j$ swapped. / $E_{type1}^{-1} = E_{type1}$

($\alpha$ $\times$ row $i$) $E_{type2}$ = $I$ with $\alpha$ in $e_{ii}$ diagonal spot / $E_{type2}^{-1}$ is $\frac{1}{\alpha}$ in $e_{ii}$ diagonal spot

[row $i$ + $m$ row $j$ = New $r_i$] $E_{type3}$ = $I$ with in $\underline{e_{ij}}$ spot put $m$ / $E_{type3}^{-1}$ is $-m$ in $e_{ij}$

ex  $\underline{\text{row } 3}$ + 4 $\underline{\text{row } 2}$ = New $r_3$  $E_{type3} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ -1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 15 & 20 & 25 \end{bmatrix}$$

So All $E_{type1,2\text{ or }3}$ are invertable (non-singular)

and $E_{type1,2\text{ or }3}^{-1}$ are still $E_{type1,2,\text{ or }3}$

**Def** A and B are called row equivalent if

$$\underbrace{E_k \cdots E_2 E_1 A}_{} = B$$

$E_i$ are elem. matrices

**Note:** $A = \underline{E_1^{-1} E_2^{-1} \cdots E_k^{-1}} B$

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**thm** three logically equivalent statements ($A$ is $n \times n$)

| Non-Singular | vs | Singular |
|---|---|---|
| (1) $A$ is non-singular | | (1) $A$ is singular |
| (2) $A\mathbf{x} = \mathbf{0}$ has <u>only trivial soln.</u> | | (2) $A\mathbf{x} = \mathbf{0}$ have non-trivial soln |
| (3) $A$ is row-equiv to $I$ | | (3) $A$ is <u>not</u> row-equiv to $I$ |

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**Note:** $A\mathbf{x} = \mathbf{b}$ vs $[A \mid b]$ or $[A \mid B]$

$EA\mathbf{x} = E\mathbf{b}$ $\leftarrow$ <u>row ops</u>

$[\quad \mid \quad]$ $\quad$ $[\quad \mid \quad]$

ex

$$\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \quad \text{vs} \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ 2 & 1 & 2 & 1 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$r_2 + (-2)r_1 = N r_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ \boxed{-2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix} \quad \text{vs} \quad \left[\begin{array}{ccc|c} 1 & -1 & 2 & 3 \\ \boxed{0} & 3 & -2 & -5 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\underline{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \boxed{-1/3} & 1 \end{bmatrix}} \underline{\begin{bmatrix} 1 & 0 & 0 \\ \boxed{-2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underbrace{\begin{bmatrix} 1 & -1 & 2 \\ 2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}}_{A} X = \text{etc}$$

aug. matrix (gauss)

$$\left[\begin{array}{c|c} \text{upper triangular} & B \end{array}\right]$$

$$\boxed{E_p \cdots E_2 E_1 A} X = E_p \cdots E_2 E_1 b$$

so

$$E_p \cdots E_2 E_1 A = U$$

$$U = \begin{bmatrix} \diagdown \\ 0 \end{bmatrix}$$

**Note:** Facts: if all $E_i$ are type 3 with the purpose of making all values below diagonal zero (gauss)

then ① $E_p \cdots E_2 E_1$ are all lower triangular

② mult. lower triangular (with 1's on diagonal) = lower triangular

③ $\underbrace{E_p \cdots E_2 E_1} A = U$

$$\boxed{A = \underbrace{(E_1^{-1} E_2^{-1} \cdots E_p^{-1})}_{\text{lower triangular}} U = L \cdot U}$$

What if we continue?

gauss

$$\left[\begin{array}{c|c} U & \end{array}\right]$$

gauss-jordan

$$\left[\begin{array}{c|c} I & \text{ans.} \end{array}\right]$$

$$\underbrace{(E_k \cdots E_2 E_1)}_{\text{must be } A^{-1}!} A = I$$

Need to find $(E_k \cdots E_2 E_1)$

trick: $[E_k \cdots E_2 E_1] I$

now: $[A \mid I]$

$[E_1 A \mid E_1 I]$

Row ops $[E_2 E_1 A \mid E_2 E_1 I]$

$[I \mid A^{-1}]$

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$$\left[\begin{array}{ccc|ccc} 1 & 4 & 3 & 1 & 0 & 0 \\ -1 & 2 & 0 & 0 & 1 & 0 \\ 2 & 2 & 3 & 0 & 0 & 1 \end{array}\right]$$

Gauss-Jordan

$[\underline{I} \mid \underline{A^{-1}}]$