Math 511

Partitioned Matrices

$A = \left[\begin{array}{cc|ccc} 1 & 2 & 1 & -1 & 0 \\ 1 & 4 & 3 & 2 & 1 \\ \hline 0 & 1 & -1 & 1 & 2 \\ \hline 2 & 1 & 1 & 3 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 2 & 1 & 3 \end{array}\right]$

6x5

$A+B = [a_{ij}+b_{ij}]$

$AB = [\vec{a}_i \, \mathbf{b}_j]$

$A = \left[\begin{array}{c|c} \overset{2\times 2}{A_{11}} & A_{12} \\ \hline A_{21} & A_{22} \\ \hline A_{31} & A_{32} \end{array}\right]$ $\quad B = \left[\begin{array}{c|c} \overset{2\times 2}{B_{11}} & B_{12} \\ \hline B_{21} & B_{22} \\ \hline B_{31} & B_{32} \end{array}\right]$

$A = \left[\begin{array}{c|c} \overset{2\times 4}{A_{11}} & \overset{2\times 6}{A_{12}} \\ \hline A_{21} & A_{22} \end{array}\right]$ $\quad B = \left[\begin{array}{c|c|c} \overset{4\times k}{B_{11}} & B_{12} & B_{13} \\ \hline \underset{6\times k}{B_{21}} & B_{22} & B_{23} \end{array}\right]$ $\quad AB$

$AB = \left[\begin{array}{c|c} \underset{2\times 4}{A_{11}} \underset{4\times k}{B_{1}} + \underset{2\times 6}{A_{12}} \underset{6\times k}{B_{21}} & \\ \hline & \end{array}\right]$

$\boxed{Q's}$ 1.5 #12c $\quad A = \;, B = \;, C =$

Solve:

$$AX + B = X$$
$$AX + B + (-1)X = X + (-1)X$$
$$AX + B - X = 0$$
$$AX - X = (-1)B$$
$$AX - IX = -B$$

$$(G+H)Q = GQ + HQ$$

$$(A-I)X = -B$$

$$(A-I)^{-1}(A-I)X = (A-I)^{-1}(-B)$$

$$X = (-1)\left[(A-I)^{-1}B\right]$$

$$A = \begin{bmatrix}5 & 3\\ 3 & 2\end{bmatrix} \quad B = \begin{bmatrix}6 & 2\\ 2 & 4\end{bmatrix}$$

$$\boxed{X = (-1)\left[\underbrace{\begin{bmatrix}4 & 3\\ 3 & 1\end{bmatrix}^{-1}}_{(A-I)}\begin{bmatrix}6 & 2\\ 2 & 4\end{bmatrix}\right]}$$

$(A-I)$

$$\left[\begin{array}{cc|cc}4 & 3 & 1 & 0\\ 3 & 1 & 0 & 1\end{array}\right] \xrightarrow{\text{row ops}} \left[\begin{array}{cc|c}1 & 0 & \\ 0 & 1 & (A-I)^{-1}\end{array}\right]$$

$$\left[\begin{array}{cc|cc}4 & 3 & 6 & 2\\ 3 & 1 & 2 & 4\end{array}\right] \xrightarrow{\text{row ops}} \left[\begin{array}{cc|c}1 & 0 & \\ 0 & 1 & \boxed{}\end{array}\right]$$

$(A-I)^{-1}B$

Ch 2

$\det(A) = 0$ then A^{-1} does not exist / A is singular

$\det(A) \neq 0$ then A^{-1} exists / A is non-singular

Basics:

$$\det\left(\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}\right) = a_{11}a_{22} - a_{12}a_{21}$$

Inductive Rule: $n \times n$

Co-factor expansions along row k

$$\det(A) = a_{k1} A_{k1} + a_{k2} A_{k2} + \cdots + a_{kn} A_{kn}$$

Co-factors: $A_{ij} = (-1)^{i+j} \left| M_{ij} \right|$ ← A's values w/o row i col j

$(n-1) \times (n-1)$

Ex:

$$\begin{vmatrix} 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 3 \\ \boxed{0 \quad 1 \quad 0 \quad -1} \\ 2 & 1 & 2 & 1 \end{vmatrix} = (+)0 \cdot \begin{vmatrix} 2 & 0 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix} + (-)(1) \begin{vmatrix} 1 & 0 & 1 \\ -1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$$(+)0 \cdot \begin{vmatrix} 1 & 2 & 1 \\ -1 & 1 & 3 \\ 2 & 1 & 1 \end{vmatrix} + (-)(-1) \begin{vmatrix} 1 & 2 & 0 \\ -1 & 1 & 2 \\ 2 & 1 & 2 \end{vmatrix}$$

$$= (-1)\left[(1) \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} + (1) \begin{vmatrix} -1 & 2 \\ 2 & 2 \end{vmatrix} \right] + \left[(1) \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} -1 & 2 \\ 2 & 2 \end{vmatrix} \right]$$

= Finish!

Cost: $\det(A)$ $n \times n$ $n!$ det that you can actually do.

Ex: 15×15 ~ 1.3 trillion det.

Note: $\det(T) = t_{11} \cdot t_{22} \cdot \ldots t_{nn}$

T ← triangular

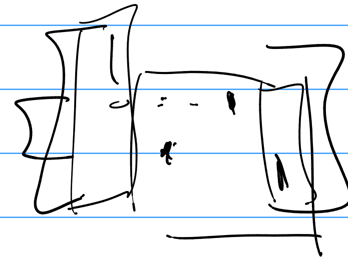
$\boxed{\text{th}^m}$ $\det(EA) = \det(E)\det(A)$

① $\det(E_{type3}) = 1$

② $\det(E_{type2}) = \alpha$

③ $\det(E_{type1}) = -1$

swap row i,j



$= \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$

So $E_k \cdots E_2 E_1 A = U$

$\det(E_k \cdots E_2 E_1 A) = \det(U)$

$\det(E_k)\cdots\det(E_2)\det(E_1)\boxed{\det(A)} = u_{11}u_{22}\cdots u_{nn}$

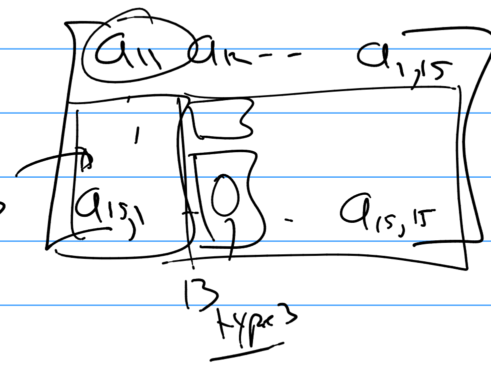
$$\boxed{\det(A) = \frac{u_{11}u_{22}\cdots u_{nn}}{\det(E_k)\cdots\det(E_2)\det(E_1)}}$$

only use type3

$$\det(A) = u_{11}u_{22}\cdots u_{nn}$$

cost?

ex 15×15



$14 + 13 + 12 + \cdots + 1$ type 3

$1 + 2 + \cdots + 14$

$\overline{15 + 15 + 15 + \cdots + 15} = \frac{15(14)}{2} = 7 \cdot 15 = 105$

by type 3 elimination

$$\begin{vmatrix} 1 & 2 & 0 & 1 \\ -1 & 1 & 2 & 3 \\ 0 & 1 & 0 & -1 \\ 2 & 1 & 2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & 2 & 4 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 2 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & 2 & 4 \\ 0 & 0 & -2/3 & -7/3 \\ 0 & 0 & 4 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 0 & 1 \\ 0 & 3 & 2 & 4 \\ 0 & 0 & -2/3 & -7/3 \\ 0 & 0 & 0 & -11 \end{vmatrix} = (1)(\cancel{3})(-2/\cancel{3})(-11)$$

$$= \boxed{22}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = -4 \begin{vmatrix} 7 & 8 & 9 \\ & 5/4 & 6/4 \\ 1 & 2 & 3 \end{vmatrix}$$

$\det\left(\begin{smallmatrix}\text{row} \\ \text{swap}\end{smallmatrix}\right) = -1$
