Math 511

Q's 2.2 #12

$$\begin{vmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ 1 & x_3 & x_3^2 \end{vmatrix} = \begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2-x_1 & x_2^2-x_1^2 \\ 0 & x_3-x_1 & x_3^2-x_1^2 \end{vmatrix}$$

$$= (1)\left[(x_2-x_1)(x_3^2-x_1^2) - (x_3-x_1)(x_2^2-x_1^2) \right]$$

$$= (x_2-x_1)(x_3-x_1)(x_3+x_1) - (x_3-x_1)(x_2-x_1)(x_2+x_1)$$

$$= (x_3-x_1)(x_2-x_1)\left[(x_3+x_1) - (x_2+x_1) \right]$$

(a) $\det(V) = \boxed{(x_3-x_1)(x_2-x_1)(x_3-x_2)}$ ←

(b) when is V non-singular? $\det(V) \neq 0$

$x_3 \neq x_1$ and $x_2 \neq x_1$ and $x_3 \neq x_2$

singular? $\det(V) = 0$

$x_3 = x_1$ or $x_2 = x_1$ or $x_3 = x_2$

$$\begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & x_2-x_1 & x_2^2-x_1^2 \\ 0 & x_3-x_1 & x_3^2-x_1^2 \end{vmatrix} = (x_2-x_1) \cdot \begin{vmatrix} 1 & x_1 & x_1^2 \\ 0 & 1 & \frac{x_2^2-x_1^2}{x_2-x_1} \\ 0 & x_3-x_1 & x_3^2-x_1^2 \end{vmatrix} = (x_3-x_1)(x_2-x_1) \begin{vmatrix} 1 & x_2 & x_1^2 \\ 0 & 1 & (x_2+x_1) \\ 0 & 1 & (x_3+x_1) \end{vmatrix}$$

type 2   type 2   etc

Exam 1   11 probs @ 10 pts

1.1 Sys. of Eqn's (1 prob) 3x3 or 4x4

① Solve w/o matrices.

1.2 2 probs ① Solve by gauss + back solve

② gauss / jordan (word problem → traffic, → ~~circuit~~)

1.3 (2 probs) Matrix Arithmetic

①, ② be able to .. $A+B$, AB, A^k, αA, A^T

given $A = \begin{bmatrix} a & 2 \\ c & \pi \end{bmatrix}$, $B = [\;]$ etc

etc $4\left[\begin{bmatrix} 1 & 2 \\ x & y \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 2 & 0 \\ -1 & 4 \end{bmatrix}^T\right] - \begin{bmatrix} 1 & 2 & 1 \\ -1 & x & y \end{bmatrix}$

1.4 Matrix Algebra (Note will use A^{-1} (1.5))
(1 prob)

① → Matrix equation → Solve for the Matrix.

ex
$$AX + B = X \quad \text{(last class?)}$$
$$AX - IX = -B$$
$$(A - I)X = -B$$
$$X = [A - I]^{-1}(-B)$$

given $A = [\;]$
$B = [\;]$

$\boxed{1.5}$ Elementary Matrices (3 probs)

① $A = LU$

$A \xrightarrow{\text{type 3 (only) row ops}} [\;\;] \rightarrow [\;\;] \rightarrow U$

$L = \begin{bmatrix} 1 & \cdot & 0 \\ +3 & \ddots & \\ +\frac{1}{2} & & 1 \end{bmatrix}$

② $A \rightarrow A^{-1}$?

③ a) know thm 1.3.1, 1.5.2 (state)

b) use those thm ideas . .

ex $A = [a_1\; a_2\; a_3]$ and $2a_1 - a_2 - 3a_3 = 0$

$$A \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix} = 0$$

$\boxed{1.6}$ $\boxed{0 \text{ probs}}$

2.1 / 2.2 (2 probs)

① $\det(A)$ → a) co-factors
→ b) elimination
→ c) interpret

② $\det(A)$ (by any method)
→ interpret.

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & 0 \\ 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & 2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & \boxed{1} & -1 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 3 & 1 & 2 \end{vmatrix}$$

Not on exam (but useful to know) 1.6, 2.3

adj A (adjoint matrix)

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & & & \\ \vdots & & & \\ a_{n1} & - & - & a_{nn} \end{bmatrix}$$

now each a_{ij} has its cofactor $A_{ij} = (-1)^{i+j} |M_{ij}|$

ex $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & 3 & 2 \end{bmatrix}$

$a_{11} = 1$ $\quad a_{23} = -1$

$A_{11} = 7$ $\quad A_{23} = -3$

$$\operatorname{adj} A = \begin{bmatrix} 7 & -2 & 3 \\ -2 & 2 & -3 \\ -1 & 1 & 1 \end{bmatrix}^T$$

$\det(A) = 5$

why?: $\underline{A}\,(\underline{\operatorname{adj} A}) = \begin{bmatrix} \det(A) & 0 & 0 \cdots \\ 0 & \det(A) & \\ 0 & & \det(A) \end{bmatrix} = \underline{\det(A)}\ \underline{I}$

$$[A]\left[\frac{1}{\det(A)} \operatorname{adj} A\right] = I$$

$$\underline{A^{-1}} = \frac{1}{\det(A)} \operatorname{adj} A$$

if $\det(A) = \pm 1$ $\quad A^{-1}$ is all integers (if A is all integers)

$$-1 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{pmatrix} = \boxed{\begin{matrix} 1 & 2 & 3 \\ 1 & 1 & 7 \\ 2 & 4 & 7 \end{matrix}}$$

type 3

"A" → Find A^{-1} = [Integers]

text: ex $\boxed{1 \quad 3 \quad 0 \quad 4 \quad 1 \quad 2 \quad 6 \quad 4 \quad 7}$ $1 \quad 1$