Math 511

Exam 1

largest issues
- #1 arithmetic / algebra
- #2 gauss / gauss-jordan
- #3 det(A)

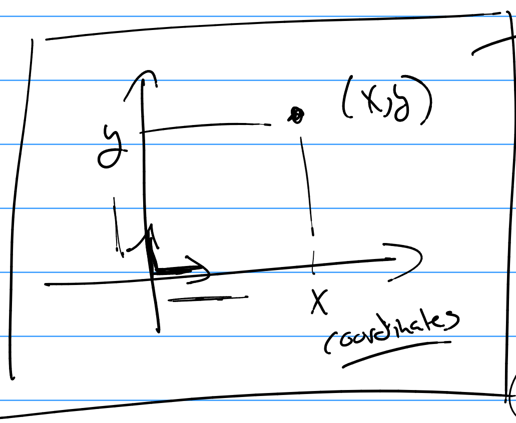
get exam back

overnight to fix for 50% missed.

Ch 3 Vector Spaces / Transformations **Ch 4**

concepts / goals

Vector Space

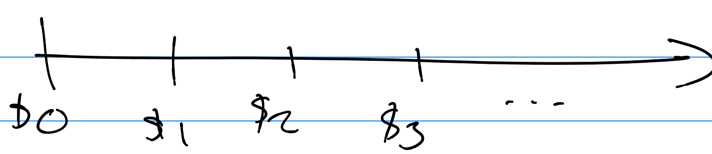


V is a set of objects

$\mathbb{R}^{2}$ ← dimension

real number

ex



$\boxed{\$3, \$5}$ ← Basis

3·$3

$0, ~~$1~~, ~~$2~~, $3, ~~$4~~, $5, $6, ~~$7~~, $8, $9, $10, ...

2·$3 $3+$5   2·$5

$\boxed{\alpha\,\$3 + \beta\,\$5} = \$n$

$Ax = b$

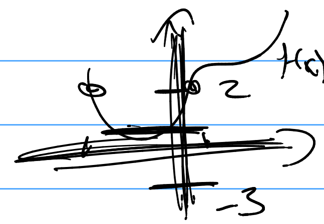
transforms:

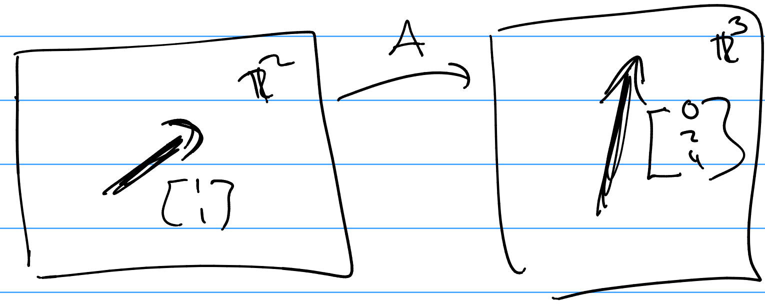
$$\underset{m\times n}{A}\ \underset{n\times 1}{x} = \underset{m\times 1}{b}$$

A is $m\times n$

ex $\underset{3\times 2}{\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix}} \underset{2\times 1}{\begin{bmatrix} 1 \\ 1 \end{bmatrix}} = \underset{3\times 1}{\begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}}$

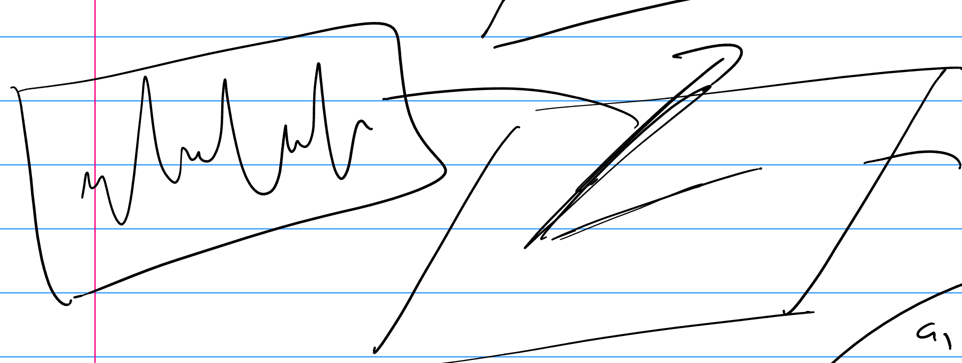
"Functions" $\begin{bmatrix} 1 \\ 1 \end{bmatrix} \xrightarrow[\text{mapping}]{A} \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$





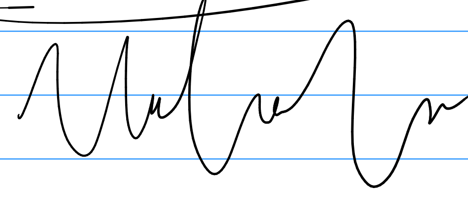
Concept:

Range is polynomials



Range is liner combos of sin(x) cos(x)

$a_1 \sin(x) + b_1 \cos(x)$
$+ a_2 \sin(2x) + b_2 \cos(2x)$
$+ a_3 \sin(3x) + b_3 \cos(3x)$



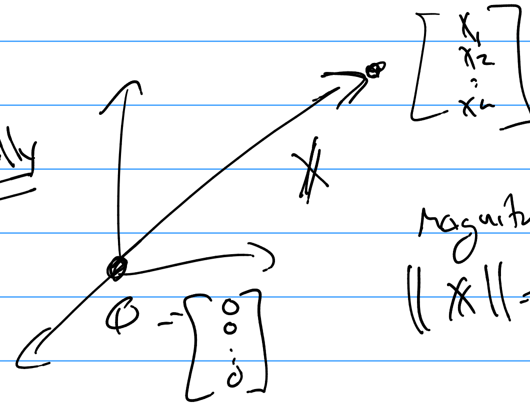
Vector Space:

$\mathbb{V}$ is a set of objects with $\mathbb{X} + \mathbb{Y}$, $\alpha \mathbb{X}$ defined.

$\mathbb{R}^n$ (n-Dimensional Space)

$$\mathbb{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

physically



magnitude

$$\|\mathbb{X}\| = \sqrt{x_1^2 + x_2^2 + \cdots + x_n^2}$$

using typical

$$\mathbb{X} + \mathbb{Y} = [x_i + y_i] \qquad \alpha \mathbb{X} = [\alpha x_i]$$

What $\boxed{\text{must}}$ be true for $\mathbb{V}$ with $\mathbb{X}+\mathbb{Y}$, $\alpha\mathbb{X}$ to be a Vector Space.

closure properties
- (1) $\mathbb{X} + \mathbb{Y} \in \mathbb{V}$ — $\in$ element of
- (2) $\alpha \mathbb{X} \in \mathbb{V}$

$\mathbb{X}+\mathbb{Y}$ properties
- A1) $\mathbb{X} + \mathbb{Y} = \mathbb{Y} + \mathbb{X}$
- A2) $(\mathbb{X} + \mathbb{Y}) + \mathbb{Z} = \mathbb{X} + (\mathbb{Y} + \mathbb{Z})$
- A3) there exists $\mathbb{0}$ such that $\mathbb{X} + \mathbb{0} = \mathbb{X}$
- A4) so $-\mathbb{X}$ exists for all $\mathbb{X}$ such that $(\mathbb{X}) + (-\mathbb{X}) = \mathbb{0}$

$\alpha\mathbb{X}$ properties
- A5) $\alpha(\mathbb{X} + \mathbb{Y}) = \alpha \mathbb{X} + \alpha \mathbb{Y}$
- A6) $(\alpha + \beta)\mathbb{X} = \alpha \mathbb{X} + \beta \mathbb{X}$
- A7) $(\alpha\beta)\mathbb{X} = \alpha(\beta\mathbb{X})$
- A8) $1 \cdot \mathbb{X} = \mathbb{X}$

Check: $\mathbb{R}^3$ with typical $\mathbb{X} + \mathbb{Y}$, $\alpha \mathbb{Y}$

in $\mathbb{R}^3$ $\quad \mathbb{X} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \mathbb{Y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \mathbb{Z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$

(1) $\mathbb{X} + \mathbb{Y} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1+y_1 \\ x_2+y_2 \\ x_3+y_3 \end{bmatrix} \in \mathbb{R}^3$ ✓

(2) $\alpha \mathbb{X} = \begin{bmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{bmatrix} \in \mathbb{R}^3$ ✓

$\vdots$

A5) $\underline{\underline{\alpha(\mathbb{X}+\mathbb{Y})}} \overset{?}{=} \alpha \mathbb{X} + \alpha \mathbb{Y}$

$$\begin{bmatrix} \alpha(x_1+y_1) \\ \alpha(x_2+y_2) \\ \alpha(x_3+y_3) \end{bmatrix} \overset{?}{=} \begin{bmatrix} \alpha x_1 \\ \alpha x_2 \\ \alpha x_3 \end{bmatrix} + \begin{bmatrix} \alpha y_1 \\ \alpha y_2 \\ \alpha y_3 \end{bmatrix} = \begin{bmatrix} \alpha x_1 + \alpha y_1 \\ \alpha x_2 + \alpha y_2 \\ \alpha x_3 + \alpha y_3 \end{bmatrix}$$

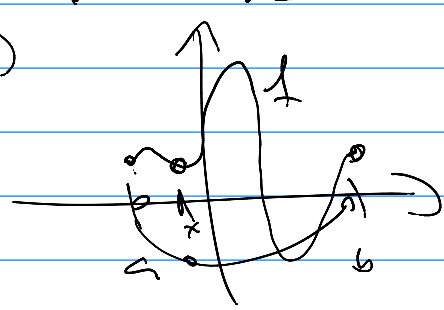
<u>yep!</u>

if you check all C_1 to A_8 and <u>all</u> hold

<u>then</u> V with $\mathbb{X}+\mathbb{Y}$, $\alpha\mathbb{X}$ is a vector space.

Ex $\quad V$ = all continuous functions over $[a,b]$

$\mathbb{X} = f(x) \quad \mathbb{Y} = g(x)$



(1) $\boxed{(f+g)(x) = f(x) + g(x)}$ ✓

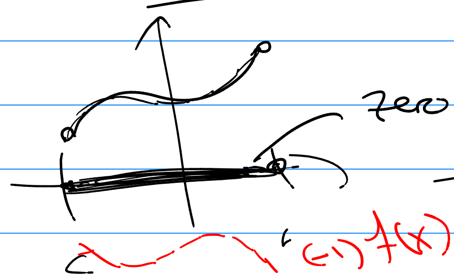
(2) $(fg)(x) = f(x)\, g(x)$ ✓

$\boxed{(\alpha f)(x) = \alpha f(x)}$ ✓

A1) $(f+g)(x) = f(x) + g(x) = g(x) + f(x) = (g+f)(x)$

A2) $(f+(g+h))(x) = f(x) + (g+h)(x)$
$= f(x) + (g(x) + h(x))$
$= (f(x) + g(x)) + h(x)$
$= ((f+g)+h)(x)$

A3) Zero function? $(f + \text{zero})(x) = f(x)$



$z(x) = 0$ ✓

A4) does $(-f)$ exist for f?

$(f + (-1)f)(x) = 0$ ✓

Vector Spaces to Know

$\alpha \mathbf{x} = \begin{bmatrix} \alpha x_1 \\ x_2 \end{bmatrix}$

objects — operations

1) $\mathbb{R}^n$ — $\mathbf{x} = [x_1, \dots, x_n]^T$ — $\mathbf{x} + \mathbf{y}$, $\alpha \mathbf{x}$ (typical)

2) $C[a,b]$ — cont. functions over $[a,b]$ — $(f+g)(x) = f(x) + g(x)$, $(\alpha f)(x) = \alpha f(x)$

3) P_n — polynomials of n-terms (degree $= n-1$) — $(p_1 + p_2)(x) = p_1(x) + p_2(x)$, $(\alpha p_1)(x) = \alpha p_1(x)$

$\mathbf{x} \to p(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_{n-1} x^{n-1}$

$\mathbf{0} \to z(x) = 0 + 0x + 0x^2 + \dots + 0x^{n-1}$

objects

ops.

(4) $\mathbb{R}^{m \times n}$ $\quad \mathcal{X} \to A = [a_{ij}]_{m \times n}$ $\qquad$ typical $\quad A+B$

αA

$\mathbb{R}^{5 \times 3}$