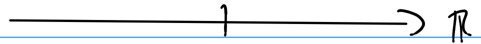
Math 511

$\boxed{Q's}$ 3.1 #13 V is a set of objects, $x+y$, αx defined and the axioms C1) C2) (closure of ops)

A1) A2) A3) A4) (Properties of $x+y$)

A5) A6) A7) A8) (properties of αx)

→ set is $\mathbb{R}$



→ $\alpha \cdot x = \alpha x$ (normal real multiplication)

$x \oplus y = \max(x,y)$

Is this a vector space means check the 10 axioms!

C1) is $x \oplus y = \max(x,y)$ a real number? yes

C2) is αx a real number? yes

A1) $x \oplus y \overset{?}{=} y \oplus x$ $\max(x,y) \overset{?}{=} \max(y,x)$ true

A2) $\underbrace{(x \oplus y) \oplus z} \overset{?}{=} \underbrace{x \oplus (\underbrace{y \oplus z})}$ true.

A3) does 0 exist? Need $x \oplus 0 = x$

$\max(x, 0) = x$ no such real number exists.

$\boxed{\text{No}}$

A4) does x have an additive inverse $(-x)$ such that $(x) + (-x) = 0$

No b/c 0 does not exist

Space $\mathbb{R}^n$ → object $v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

Space $\mathbb{R}^{m\times n} \rightarrow$ object $V = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & - & v_{2n} \\ \vdots & & & \\ v_{m1} & - & \cdots & v_{mn} \end{bmatrix}$

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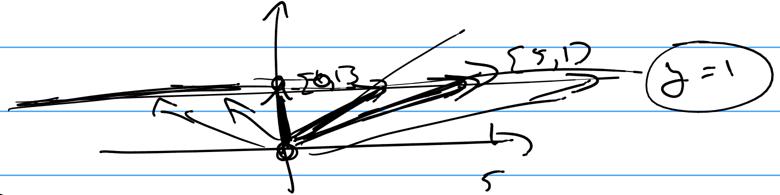
p116 example   Set of objects are $\begin{bmatrix} a \\ 1 \end{bmatrix}$

(typical $v_1 + v_2$, αv_1)

ex



closure: $\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}^{5} \notin W$ (not closed)

3.2 Subspace: given V a vector space and consider a subset of objects in V, called S.

$$S \subseteq V$$

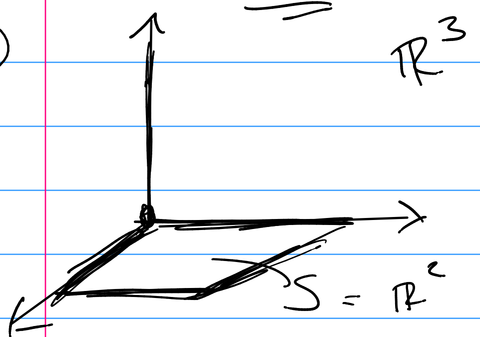
and we see

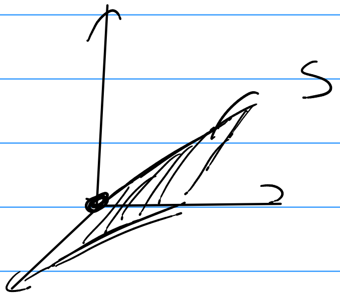
① S is non-empty
② $0 \in S$
③ $v_1, v_2 \in S$ $v_1 + v_2 \in S$
④ $\alpha v_1 \in S$

So S is a vector space that is also $S \subseteq V$

Call S a subspace of V.

ex



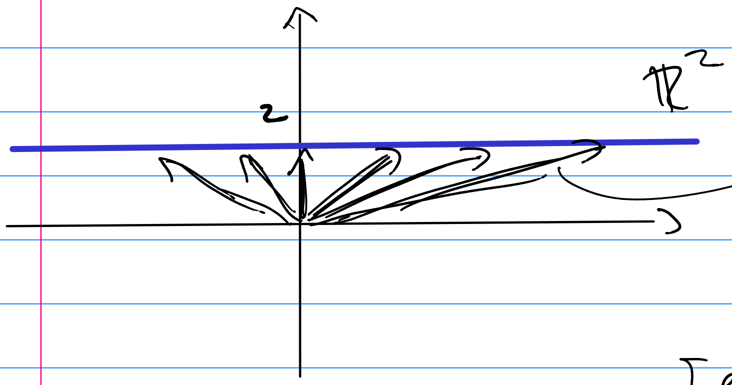


to check if S is a subspace we need ...

① $\mathbf{0} \in S$?

② is $v_1 + v_2$ closed?

③ is αv_1 closed?



$$S = \left\{ \begin{bmatrix} \alpha \\ 2 \end{bmatrix} \;\middle|\; \alpha \in \mathbb{R} \right\}$$

check: ① $\mathbf{0} \in S$?

$\begin{bmatrix} 0 \\ 0 \end{bmatrix} \in S$ $\boxed{\text{no}}$

② $v_1 + v_2 = \begin{bmatrix} \alpha_1 \\ 2 \end{bmatrix} + \begin{bmatrix} \alpha_2 \\ 2 \end{bmatrix} = \begin{bmatrix} \alpha_1 + \alpha_2 \\ 4 \end{bmatrix} \notin S$

③ $\alpha v_1 = \begin{bmatrix} \alpha \alpha_1 \\ \alpha 2 \end{bmatrix} \notin S$ if $\alpha \neq 1$

$\mathbb{R}^{2\times 2}$ any $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ matrix

S is all 2x2 matrices such that $d = 0$

$$S = \left\{ \begin{bmatrix} a & b \\ c & \underset{\uparrow}{0} \end{bmatrix} \;\middle|\; a, b, c \in \mathbb{R} \right\}$$

check ① is $\mathbf{0} \in S$?

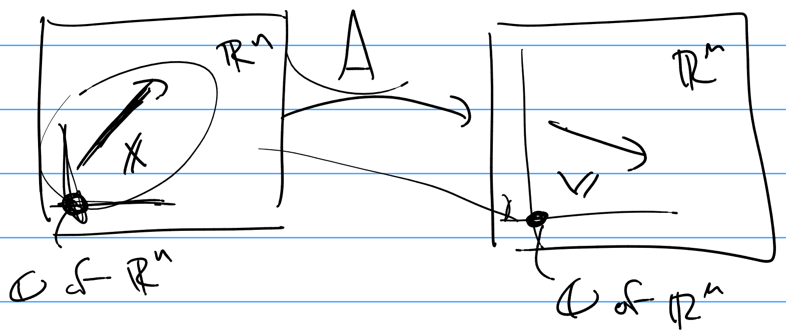
$\begin{bmatrix} 0 & 0 \\ 0 & \boxed{0} \end{bmatrix} \in S$ yes!

② $\begin{bmatrix} a & b \\ c & 0 \end{bmatrix} + \begin{bmatrix} d & e \\ f & 0 \end{bmatrix} = \begin{bmatrix} a+d & b+e \\ c+f & \boxed{0} \end{bmatrix} \in S$ yes!

③ $\alpha \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} = \begin{bmatrix} \alpha a & \alpha b \\ \alpha c & \boxed{0} \end{bmatrix} \in S$ yes!

so S is a subspace of $\mathbb{R}^{2\times 2}$

Consider A as a transform (A is $m \times n$)



$$\underset{m\times n}{A}\,\underset{n\times 1}{x} = \underset{m\times 1}{v}$$

$$\underline{Ax = 0}$$

collect all x such that $Ax = 0$

$N(A) = \{x \mid Ax = 0\}$ <u>Null</u> space of A

<u>Note:</u> $N(A)$ IS a subspace of $\mathbb{R}^n$

check ① $0 \in N(A)$ <u>yes!</u> b/c $A \underset{\text{of } \mathbb{R}^n}{0} = \overset{\text{of } \mathbb{R}^m}{0}$

② if $\underline{\underline{x_1}}, \underline{\underline{x_2}} \in N(A)$ is $\underline{\underline{x_1 + x_2}}$?

$$A(x_1 + x_2) = Ax_1 + Ax_2 = 0 + 0 = 0$$

yes!

③ is αx_1 in $N(A)$?

$$A(\alpha x_1) = \alpha[Ax_1] = \alpha 0 = 0$$

yes!

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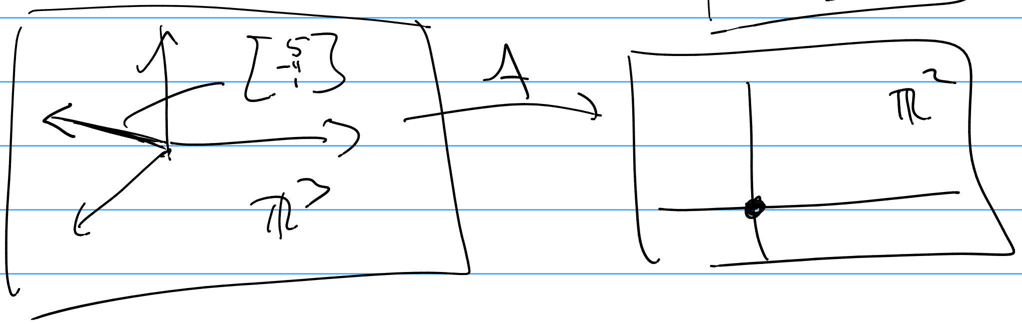
ex $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \end{bmatrix}$ $\underline{\underline{N(A) = ?}}$

Soln's to $Ax = 0$ $\left[\begin{array}{ccc|c} \textcircled{1} & \textcircled{2} & \textcircled{3} & 0 \\ 0 & \textcircled{1} & \underset{\uparrow\,\text{free}}{4} & 0 \end{array}\right]$

$x_3 = \alpha$ $x_2 = -4\alpha$ $x_1 = 5\alpha$

$$\mathbb{X} = \begin{bmatrix} 5\alpha \\ -4\alpha \\ \alpha \end{bmatrix} = \alpha \boxed{\begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}}$$



Def any linear combination of $\mathbb{V}_1, \mathbb{V}_2, \dots, \mathbb{V}_k$

$$\mathbb{V} = \alpha_1 \mathbb{V}_1 + \alpha_2 \mathbb{V}_2 + \dots + \alpha_k \mathbb{V}_k$$

Set of all linear combos is called the Span $(\mathbb{V}_i)$

ex $\mathbb{V}_1 = \$3$ $\mathbb{V}_2 = \$5$

$$\$a = \alpha_1 \boxed{\$3} + \alpha_2 \boxed{\$5}$$

$$\text{Span}(\$3, \$5) = \$0, \$3, \$5, \$6, \$8, \$9, \$10, \$11, \dots$$

from above $N(A) = \text{Span}\left(\begin{bmatrix} 5 \\ -4 \\ 1 \end{bmatrix}\right)$