Math 511

Q's P_4 , 4 term polynomial $p(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

P_n

→ Vector version: $\mathbb{P} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}$

or $p(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$

→ Vector version $\mathbb{P} = \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$ ← x^3, ← x^2, ← x, ← const.

3.R #11d Spanning Sets

$\text{Span}(\{v_1, v_2, \ldots, v_k\}) = \{ v \mid c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = v \}$

Note: #1 $\text{Span}(\{v_1, v_2, \ldots, v_k\})$ subspace of V

#2 Spanning Set : $\text{Span}(\ ) = V$

Says: we always have some c_i & v_i for any $v \in V$

So Solve $c_1 v_1 + c_2 v_2 + \cdots + c_k v_k = \boxed{\text{represent}}$ all possible $v \in V$

has a soln → spanning set

do not always have a soln → not a spanning set.

11d is $\begin{bmatrix}-1\\2\end{bmatrix}, \begin{bmatrix}1\\-2\end{bmatrix}, \begin{bmatrix}2\\-4\end{bmatrix}$ spanning set $\xrightarrow{}$ any $\vec{v}$ in $\mathbb{R}^2$

(of $\mathbb{R}^2$

$$c_1\begin{bmatrix}-1\\2\end{bmatrix} + c_2\begin{bmatrix}1\\-2\end{bmatrix} + c_3\begin{bmatrix}2\\-4\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$$

Solve
$$\left[\begin{array}{ccc|c}-1 & 1 & 2 & a\\ 2 & -2 & -4 & b\end{array}\right]$$

$$\left[\begin{array}{ccc|c}-1 & 1 & 2 & a\\ 0 & 0 & 0 & 2a+b\end{array}\right]$$

$0 = 2a+b$ no soln.

ex of one with a soln. $\begin{bmatrix}-2\\1\end{bmatrix}, \begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}2\\4\end{bmatrix} \in \mathbb{R}^2$

$$c_1\begin{bmatrix}-2\\1\end{bmatrix} + c_2\begin{bmatrix}1\\3\end{bmatrix} + c_3\begin{bmatrix}2\\4\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$$

Solve:
$$\left[\begin{array}{ccc|c}-2 & 1 & 2 & a\\ 1 & 3 & 4 & b\end{array}\right] \to \left[\begin{array}{ccc|c}1 & 3 & 4 & b\\ -2 & 1 & 2 & a\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1 & 3 & 4 & b\\ 0 & 7 & 10 & a+2b\end{array}\right]$$

c_3 is free

$c_3 = \alpha \to c_2 = \dfrac{a+2b-10\alpha}{7}$

$\to c_1 = b - 4\alpha - \frac{3}{7}(a+2b-10\alpha)$

(i) has a soln.
So spanning set

$$c_1\begin{bmatrix}-2\\1\end{bmatrix} + c_2\begin{bmatrix}1\\3\end{bmatrix} + c_3\begin{bmatrix}2\\4\end{bmatrix} = \begin{bmatrix}a\\b\end{bmatrix}$$

$$-\frac{2}{7}\begin{bmatrix}-2\\1\end{bmatrix} + \frac{3}{7}\begin{bmatrix}1\\3\end{bmatrix} + 0\begin{bmatrix}2\\4\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$$

Note: in a problem with free variables...

ex

$$\begin{array}{cccc} \mathbb{V}_1 & \mathbb{V}_2 & \mathbb{V}_3 & \mathbb{V}_4 \\ \begin{bmatrix}1\\0\\0\end{bmatrix}, & \begin{bmatrix}1\\1\\1\end{bmatrix}, & \begin{bmatrix}2\\1\\1\end{bmatrix}, & \begin{bmatrix}2\\-1\\-1\end{bmatrix} \end{array}$$

doing spans:

$$c_1\begin{bmatrix}1\\0\\0\end{bmatrix} + c_2\begin{bmatrix}1\\1\\1\end{bmatrix} + c_3\begin{bmatrix}2\\1\\1\end{bmatrix} + c_4\begin{bmatrix}2\\-1\\-1\end{bmatrix} = \mathbb{V}$$

Notice:

$$\mathbb{V}_1 + \mathbb{V}_2 = \mathbb{V}_3$$
$$2\mathbb{V}_1 - \mathbb{V}_2 = \mathbb{V}_4$$

how do we see this?

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 2 & \\ 0 & 1 & 1 & -1 & \\ 0 & 1 & 1 & -1 & \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 1 & 2 & 2 & \\ 0 & 1 & 1 & -1 & \\ 0 & 0 & 0 & 0 & \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 3 & \\ 0 & 1 & 1 & -1 & \\ 0 & 0 & 0 & 0 & \end{array}\right]$$

free free

$$\boxed{\begin{aligned} \mathbb{V}_3 &= \mathbb{V}_1 + \mathbb{V}_2 \\ \mathbb{V}_4 &= 3\mathbb{V}_1 - \mathbb{V}_2 \end{aligned}}$$

linear dependence means some $\mathbb{V}_i$ is a linear combo of the other vectors.

Consider that $\mathbb{V}_3 = \mathbb{V}_1 + \mathbb{V}_2$

$$\rightarrow (1)\mathbb{V}_1 + (1)\mathbb{V}_2 + (-1)\mathbb{V}_3 + (0)\mathbb{V}_4 = \mathbb{0}$$

$$\mathbb{V}_4 = 3\mathbb{V}_1 - \mathbb{V}_2$$

$$\rightarrow (3)\mathbb{V}_1 + (-1)\mathbb{V}_2 + (0)\mathbb{V}_3 + (-1)\mathbb{V}_4 = \mathbb{0}$$

non-trivial soln to homogeneous system

Def $\mathbb{V}_1, \mathbb{V}_2, \dots, \mathbb{V}_k$ in $\mathbb{V}$ are linearly dependent if

$$\boxed{c_1 \mathbb{V}_1 + c_2 \mathbb{V}_2 + \dots + c_k \mathbb{V}_k = \mathbb{O}}$$

has non-trivial soln's.

Def linearly independent if $\underline{\underline{\text{only}}}$ trivial soln.

Note: if $\mathbb{V} = \mathbb{R}^n$ and $\mathbb{V}_1, \mathbb{V}_2, \dots, \mathbb{V}_n \in \mathbb{R}^n$ $\mathbb{V} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

$$c_1 \mathbb{V}_1 + c_2 \mathbb{V}_2 + \dots + c_n \mathbb{V}_n = \mathbb{O}$$

$$\underbrace{\boxed{\begin{bmatrix} \mathbb{V}_1 & \mathbb{V}_2 & \cdots & \mathbb{V}_n \end{bmatrix}}}_{n \times n} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \mathbb{O}$$

linearly ind. is only trivial soln but b/c matrix is $n \times n$ we can use $\det(\ )$

$$\det([\mathbb{V}_1\ \mathbb{V}_2 \cdots \mathbb{V}_n]) = 0 \to \text{dep}$$
$$\neq 0 \to \text{indep.}$$

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test $\mathbb{V}_1, \mathbb{V}_2, \dots, \mathbb{V}_k$ for dep/ind, you have to setup and solve $c_1 \mathbb{V}_1 + c_2 \mathbb{V}_2 + \dots + c_k \mathbb{V}_k = \mathbb{O}$ ←

Great property of linearly independent vectors

↳ consider span $c_1 \mathbb{V}_1 + c_2 \mathbb{V}_2 + \dots + c_k \mathbb{V}_k = \boxed{\mathbb{V}}$

the c_i are unique to get to $\mathbb{V}$.

b/c <u>uniqueness</u> we can use a coordinate

notation $v = \begin{bmatrix} c_1 \\ c_2 \\ c_k \end{bmatrix}_{\{v_i\}}$

set of v_i's

<u>coordinates</u>