

$$P_4 \rightarrow ax^3 + bx^2 + cx + d \quad \text{Zero: } 0 + 0x + 0x^2 + 0x^3 = z(x)$$

$$S = \{ p \mid p \text{ is of odd degree} \}$$

means?

$$\deg(x^3) = 3$$

$$\deg(p) = \text{highest power}$$

$$\deg(x^3 + x^2 + 2) = 3$$

$$\deg(x^2 + x + 1) = 2$$

$$\deg(x^4 + 2x^3 + x) = 4$$

$$\deg(p) = \text{odd?}$$

take all  $p$  whose highest power is odd number

$$\deg(x^3 + x^2 + 2) = 3$$

$$3 = 3 \cdot x^0$$

$$\deg(x^1 - 1) = 1$$

$$\deg(x^2 + 2) = 2$$

$$\deg(x^0) = 0$$

$$\deg(0) = 0$$

check subspace:

$$\text{\#1 } 0 \in S? \quad \deg(0 + 0x + 0x^2 + 0x^3) = 0$$

act on

Not a subspace

deg.

\#2  $p_1 + p_2$  closed?

$$(x^3) + (-x^3 + x^2) = x^2$$

$$\text{Span}(x_1, x_2) \rightarrow \forall v = c_1 x_1 + c_2 x_2$$

$$\underline{\underline{13x}} \quad \begin{bmatrix} 2 \\ 6 \\ 6 \end{bmatrix} \stackrel{?}{=} c_1 \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$

Spanning set  $c_1 v_1 + c_2 v_2 + c_3 v_3 =$  everywhere

(P3)  $c_1(2) + c_2(x^2) + c_3(x) + c_4(2x+3) = a + bx + cx^2$

<u>const:</u>	$2c_1 + 3c_4 = a$	}	<u>solve:</u>	$c_1 = ?$
<u>x:</u>	$c_3 + 2c_4 = b$		$c_2 = ?$	
<u>x<sup>2</sup>:</u>	$c_2 = c$		$c_3 = ?$	
				$c_4 = ?$

$$\left[ \begin{array}{cccc|c} 2 & 0 & 0 & 3 & a \\ 0 & 0 & 1 & 2 & b \\ 0 & 1 & 0 & 0 & c \end{array} \right] \rightarrow \left[ \begin{array}{cccc|c} 2 & 0 & 0 & 3 & a \\ 0 & 1 & 0 & 0 & c \\ 0 & 0 & 1 & 2 & b \end{array} \right]$$

↑  
free

has a soln so yes, spanning

(P3)

$$p_1 = 2 \quad p_2 = 1 + x^2 \quad p_3 = 3x^2$$

$$c_1(2) + c_2(1 + x^2) + c_3(3x^2) = a + bx + cx^2$$

<u>const:</u>	$2c_1 + c_2 = a$	}	no soln
<u>x:</u>	$0 = b$		
<u>x<sup>2</sup>:</u>	$c_2 + 3c_3 = c$		

$$\left[ \begin{array}{ccc|c} 2 & 1 & 0 & a \\ 0 & 0 & 0 & b \\ 0 & 1 & 3 & c \end{array} \right]$$

# Spanning Sets

$$P_4 \implies a + bx + cx^2 + dx^3$$
$$\mathbb{R}^{2 \times 3} \rightarrow \left\{ \begin{array}{l} a_{11} \ a_{12} \ a_{13} \\ a_{21} \ a_{22} \ a_{23} \end{array} \right\}$$

$$c_1 v_1 + c_2 v_2 + \dots + c_k v_k = \boxed{\text{everywhere}}$$