Math 511

Q's 3.4 #5 $\mathbf{x}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $\mathbf{x}_2 = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$ $\mathbf{x}_3 = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$

a) show $\begin{vmatrix} 2 & 3 & 2 \\ 1 & -1 & 6 \\ 3 & 4 & 4 \end{vmatrix} = 0$

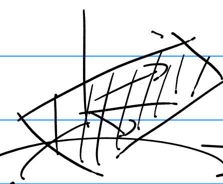
b) $\mathbf{x}_1 = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $\mathbf{x}_2 = \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$ show lnd.

$\left[\begin{array}{cc|c} 2 & 3 & 0 \\ 1 & -1 & 0 \\ 3 & 4 & 0 \end{array}\right] \longleftarrow$ solve $c_1 \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

or subs or elim

has only trivial soln.

c) $\dim(\mathrm{Span}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)) = 2$



$\mathrm{Span}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \mathrm{Span}(\mathbf{x}_1, \mathbf{x}_2)$ plane in $\mathbb{R}^3$

Change of Basis

Idea: units $[10]_{in} \xrightarrow{?} [\ \ ]_{cm}$

use the fact that $1 \cdot X = X$

know $2.54\,cm = 1\,in \rightarrow \frac{2.54\,cm}{1\,in} = \frac{1\,in}{2.54\,cm} = 1$

$\left[\frac{1\,in}{2.54\,cm}\right][8]_{cm} = \left[\frac{8}{2.54}\right]_{in} \approx [3.15]_{in}$

$\left[\frac{2.54\,cm}{1\,in}\right][10]_{in} = [25.4]_{cm}$

$\left[\text{in to cm}\right][\ \ ]_{in} = [\ \ ]_{cm}$

$[\quad]_{in} = \begin{bmatrix} \text{in to cm} \end{bmatrix}^{-1} [\quad]_{cm}$

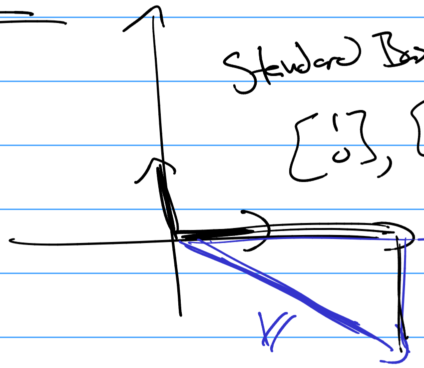
"$= \begin{bmatrix} \text{cm to in} \end{bmatrix}$"

Vector Space?

ex $\mathbb{R}^2$

Standard Basis

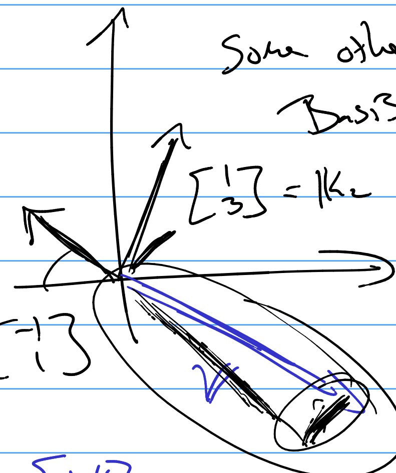
$\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}$



Some other Basis

$\begin{bmatrix}1\\3\end{bmatrix} = k_2$

$k_1 = \begin{bmatrix}-1\\1\end{bmatrix}$



$V = [V]_E$

coord in standard

$V = \alpha_1 e_1 + \alpha_2 e_2$

$[V]_E = \begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix}$

$V = [V]_K$

coord in basis k_i

$V = \beta_1 k_1 + \beta_2 k_2$

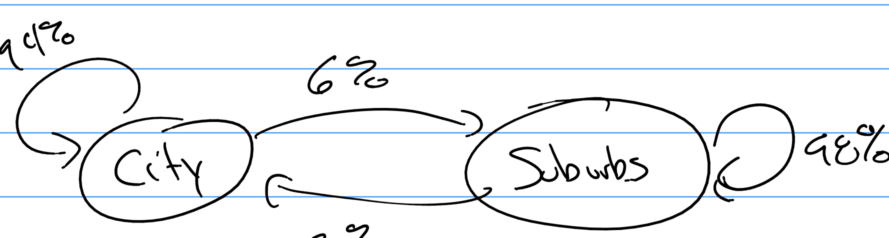
$[V]_K = \begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix}$

before "how to convert" $[V]_E \longrightarrow [V]_K$?

or $[V]_K \longrightarrow [V]_E$?

1st is why?

ex



$X = \begin{bmatrix} \text{\% of pop in city} \\ \text{\% of pop in sub} \end{bmatrix}$

$A = \begin{array}{cc} \text{city} & \text{sub} \\ \end{array}$

$A = \begin{bmatrix} .94 & .02 \\ .06 & .98 \end{bmatrix} \begin{array}{l} \text{to city} \\ \text{to sub} \end{array}$

Markov Process / Markov Chain

$AX = \begin{bmatrix} \text{city to city} & \text{sub to city} \\ \text{city to sub} & \text{sub to sub} \end{bmatrix} \begin{bmatrix} \text{city} \\ \text{sub} \end{bmatrix} = \begin{bmatrix} \text{city} \\ \text{sub} \end{bmatrix}$

$\mathbb{X}_0$ is start pop.

$$\mathbb{X}_1 = A\mathbb{X}_0, \quad \mathbb{X}_2 = A\mathbb{X}_1 = A(A\mathbb{X}_0) = A^2\mathbb{X}_0, \ldots$$

So $\mathbb{X}_k = A\mathbb{X}_{k-1}$ but $\mathbb{X}_k = A^k\mathbb{X}_0$

Markov process

Markov chain $\mathbb{X}_0, \mathbb{X}_1, \mathbb{X}_2, \mathbb{X}_3, \ldots$

$$A = \begin{bmatrix} .94 & .02 \\ .06 & .98 \end{bmatrix}$$ "Notice"

Found them

$$A\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}1\\3\end{bmatrix} \qquad A\begin{bmatrix}-1\\1\end{bmatrix} = .92\begin{bmatrix}-1\\1\end{bmatrix}$$

$$A\mathbb{K}_2 = \mathbb{K}_2 \qquad A\mathbb{K}_1 = .92\,\mathbb{K}_1$$

$$\mathbb{V} = \beta_1\mathbb{K}_1 + \beta_2\mathbb{K}_2$$

$$A\mathbb{V} = A(\beta_1\mathbb{K}_1 + \beta_2\mathbb{K}_2) = \beta_1 A\mathbb{K}_1 + \beta_2 A\mathbb{K}_2$$

$$A\mathbb{V} = (.92)\beta_1\mathbb{K}_1 + \beta_2\mathbb{K}_2$$

So $A^n\mathbb{V} = \cancel{(.92)^n\beta_1\mathbb{K}_1}^{\to 0} + \beta_2\mathbb{K}_2$

So some basis vectors are more useful for some applications (like Markov chains)

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change of basis

$$A\mathbf{x} = \mathbf{b}$$

$$A\mathbf{x} = [\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3 \cdots \mathbf{a}_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$A\mathbf{x} = \boxed{x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + \cdots + x_n\mathbf{a}_n}$$

so

coord in Basis $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n$ let $B = [\mathbf{b}_1\ \mathbf{b}_2 \cdots \mathbf{b}_n]$

$$\mathbf{v} = c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + \cdots + c_n\mathbf{b}_n$$

$$B\begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix} = \boxed{B[\mathbf{v}]_B}$$

coord in basis B

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so any $\mathbf{v}$ can be represented in a basis using

ex: Basis $\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n \rightarrow B = [\mathbf{b}_1\ \mathbf{b}_2 \cdots \mathbf{b}_n]$

$\mathbf{v}$

$$\mathbf{v} = B[\mathbf{v}]_B$$

but: standard basis $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n$ has $I = [\mathbf{e}_1\ \mathbf{e}_2 \cdots \mathbf{e}_n]$

$$\mathbf{v} = [\mathbf{v}]_E$$

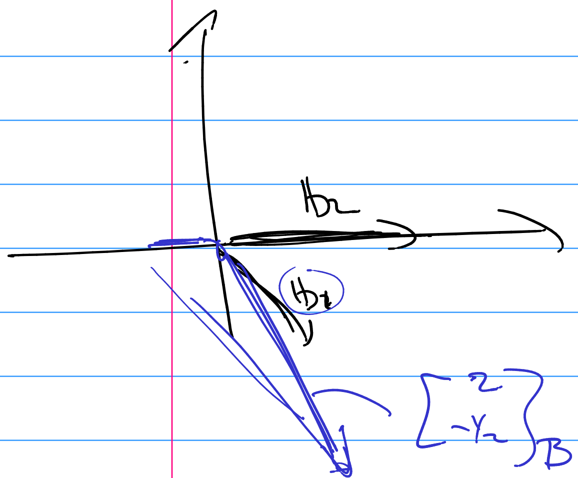
so

b/c $\mathbf{v} = \mathbf{v}$

$$[\mathbf{v}]_E = B[\mathbf{v}]_B$$

and $\quad [v]_B = B^{-1}\,[v]_E$

Ex $\quad$ Basis $\quad b_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \quad b_2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$



$2 b_1 + -\frac{1}{2} b_2 = \begin{bmatrix} 2 \\ -1/2 \end{bmatrix}_B$

$[v]_E = \begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ -1/2 \end{bmatrix}_B$

$= \begin{bmatrix} 1 \\ -4 \end{bmatrix}_E$

$\begin{bmatrix} 1 & 2 \\ -2 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 3 \\ 7 \end{bmatrix}_E = \begin{bmatrix} \quad \end{bmatrix}_B$