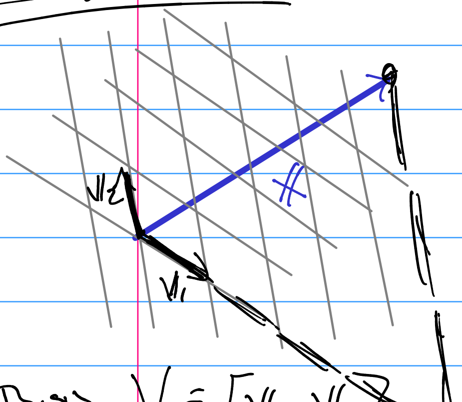
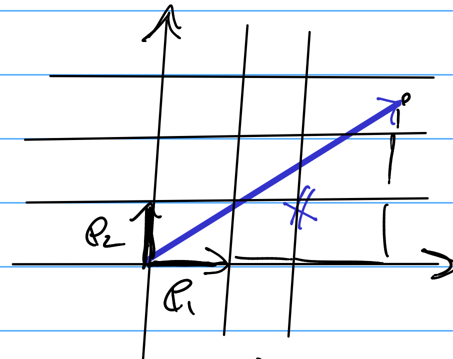
Math 511

Change of Basis



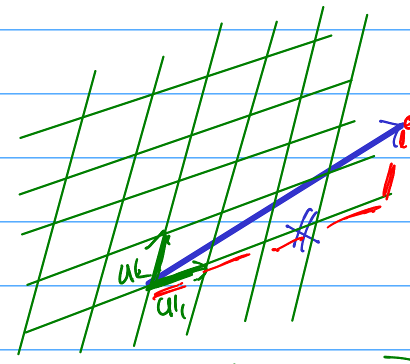
Basis $V = [v_1 \ v_2]$

$[x]_V$



standard

x or $[x]_E$



Basis $U = [u_1, \ u_2]$

$[x]_U$

Know: #1 — $V[x]_V = [x]_E$

$U[x]_U = [x]_E$

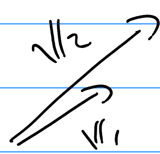
#2 $\boxed{V[x]_V = U[x]_U}$

so $[x]_V = V^{-1}U[x]_U$ transition matrix from U to V

$[x]_U = U^{-1}V[x]_V$ transition matrix from V to U

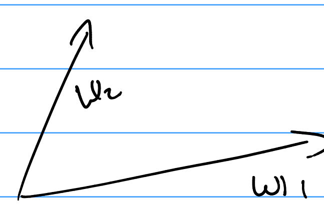
ex

3.5 #7



$v_1 = \begin{bmatrix}1\\2\end{bmatrix}$ $v_2 = \begin{bmatrix}2\\3\end{bmatrix}$

$V = \begin{bmatrix}1 & 2\\2 & 3\end{bmatrix}$



$W = [w_1 \ w_2]$

Say: that from $[X]_W$ to $[X]_V$ is ..

$$V[X]_V = W[X]_W$$

$$[X]_V = V^{-1}W [X]_W$$

$$\shortparallel$$

from 3.5 #7 say $S = \begin{bmatrix} 3 & 5 \\ 1 & -2 \end{bmatrix}$

$$\begin{bmatrix} 3 & 5 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}^{-1} W$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 1 & -2 \end{bmatrix} = W$$

$$\begin{bmatrix} 5 & 1 \\ 9 & 4 \end{bmatrix} = W \qquad \text{so } w_1 = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$$

$$w_2 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

P_n as $\mathbb{R}^n$

representing polynomials in P_n as vectors in $\mathbb{R}^n$

#1 decide on order of polynomial .. (individual choice)

ex $P_5 \to p(x) = a + bx + cx^2 + dx^3 + ex^4$

write $p(x)$ as ... $\mathbb{P} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}$

ex $p_1(x) = 2x - x^3 \to \mathbb{P}_1 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \\ 0 \end{bmatrix} \in \mathbb{R}^5$

#2 test (for P_n) $p_1, p_2, \dots, p_k$ for ind / dependence / etc

use: $\boxed{\mathbb{P}_1, \mathbb{P}_2, \dots, \mathbb{P}_k \text{ in } \mathbb{R}^n}$

any $p(x) = a + bx + cx^2$

ex: P_3 are $p_1(x) = 1+x$ $\quad p_2(x) = 3-x^2$ $\quad p_3(x) = x+x^2$

dep or ind?

test: $\mathbb{P}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ $\quad \mathbb{P}_2 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ $\quad \mathbb{P}_3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

3 vectors of $\mathbb{R}^3$ so.. use det( )

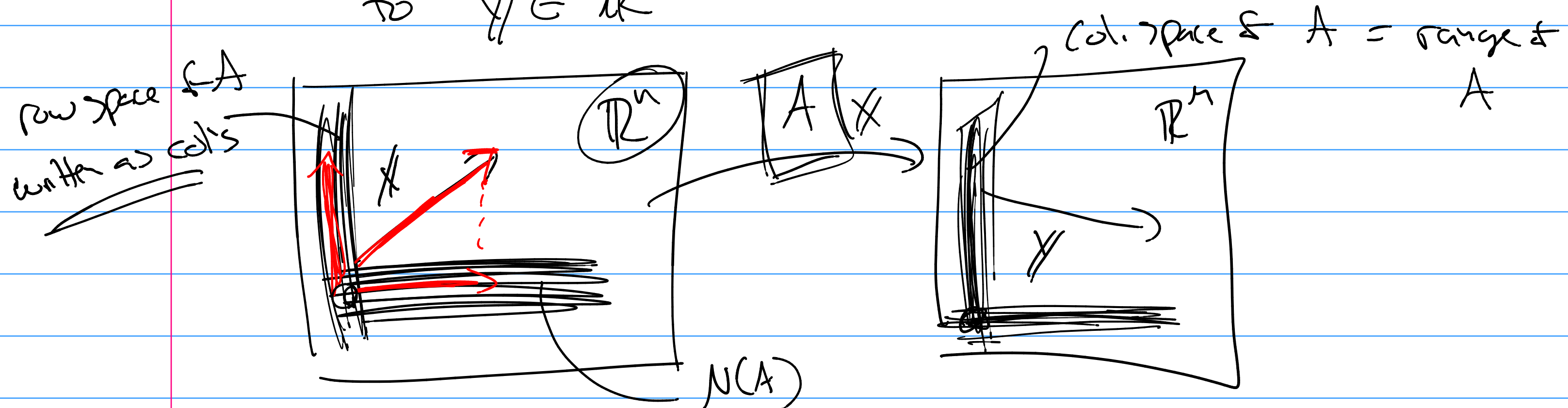
$$\begin{vmatrix} 1 & 3 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 1 \\ -1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 0 \\ -1 & 1 \end{vmatrix}$$

$$= (1) - (3) = -2$$

non-zero det so $\boxed{\text{ind,}}$

3.6 of $A_{m \times n}$ $\qquad \underset{m\times n}{A} \, \underset{n\times 1}{x} = \underset{m\times 1}{y}$

we can consider A as a function moving $x \in \mathbb{R}^n$ to $y \in \mathbb{R}^m$



to study A as a transform we need to be able to take A

$$A \xrightarrow[\text{ops}]{\text{row}} U_{gauss} \xrightarrow[\text{ops}]{\text{row}} U_{gauss/jordan}$$

$$\begin{bmatrix} \boxed{1} & 2 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 1 & -1 & 0 & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & -3 & 1 & -1 \end{bmatrix} \to \underbrace{\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 7 & -4 \end{bmatrix}}_{U_{gauss}}$$

$\frac{15}{7}$ $\frac{1}{7}$ $-\frac{4}{7}$

① we could state x_1, x_2, x_3 are leads

x_4 is free

② solve $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 7 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ to get $N(A) = \text{span}(\quad)$

③ Continue:

$$\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 7 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 1 & -4/7 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 0 & 17/7 \\ 0 & 1 & 0 & 1/7 \\ 0 & 0 & 1 & -4/7 \end{bmatrix}$$

$$\to \underbrace{\begin{bmatrix} 1 & 0 & 0 & 15/7 \\ 0 & 1 & 0 & 1/7 \\ 0 & 0 & 1 & -4/7 \end{bmatrix}}_{U}$$

$$u_4 = \frac{15}{7} u_1 + \frac{1}{7} u_2 - \frac{4}{7} u_3$$

so

$$a_4 = \frac{15}{7} a_1 + \frac{1}{7} a_2 - \frac{4}{7} a_3$$

dep. eqn(s)

3.6 $A = [a_1, a_2 \dots a_n] = \begin{bmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_n \end{bmatrix}$

#1 take any linear combo of a_i

$$v = c_1 a_1 + c_2 a_2 + \dots + c_n a_n$$

$$\text{Column Space of } A = \{ v \mid v \in \text{Span}(a_1, \dots a_n) \}$$

#2 row space of A, linear combo of $\vec{a_i}$,

row space of A $= \{ \vec{v} \mid \vec{v} = c_1\vec{a}_{1} + c_2\vec{a}_{2} + \dots + c_n\vec{a}_{n} \}$