Math 511

$\boxed{\text{Q's}}$ 3.3 #7 $\quad x_1, x_2, x_3$ are lin. indep.

if $\quad y_1 = x_2 - x_1 \qquad y_2 = x_3 - x_2 \qquad y_3 = x_3 - x_1$

are the y's lin ind / dep?

use check ← solve $c_1, c_2, c_3 = ?$

$$c_1 y_1 + c_2 y_2 + c_3 y_3 = 0 \quad \leftarrow \text{lin. combo of } y\text{'s}$$

$$c_1(x_2 - x_1) + c_2(x_3 - x_2) + c_3(x_3 - x_1) = 0$$

$$\boxed{(-c_1 - c_3)x_1 + (c_1 - c_2)x_2 + (c_2 + c_3)x_3 = 0} \quad \leftarrow \text{lin combo of } x\text{'s}$$

b/c x's are ind the system is

$$\begin{aligned} -c_1 - c_3 &= 0 \\ c_1 - c_2 &= 0 \\ c_2 + c_3 &= 0 \end{aligned} \qquad \begin{aligned} -c_1 - c_3 &= 0 \\ c_1 + c_3 &= 0 \\ \hline 0 &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

↑ free!

non-trivial sol's

so $\boxed{\text{lin. dep}}$

Or you could have "seen"..

$$\left.\begin{aligned} y_1 &= x_2 - x_1 \\ y_2 &= x_3 - x_2 \end{aligned}\right\} \quad y_1 + y_2 = x_3 - x_1 = y_3$$

lin. dep.

3.4 #14 dimension of ..

Note: $\dim(\text{span}(v_1, v_2, \dots, v_k))$ $= \#$ of ind. vectors

(a) $\text{Span}(x, x-1, x^2+1)$

$\boxed{\dim = 3}$

$\boxed{P_3 \to \mathbb{R}^3 \quad p(x) = a + bx + cx^2}$

$$\text{Span}\left(\begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}\right)$$

$$\begin{bmatrix}0 & -1 & 1\\1 & 1 & 0\\0 & 0 & 1\end{bmatrix} \to \begin{bmatrix}1 & 1 & 0\\0 & -1 & 1\\0 & 0 & 1\end{bmatrix}$$

(b) $\text{Span}(x, x-1, x^2+1, \cancel{x^2-1})$ $\dim = 3$

check:
$$\begin{bmatrix}0 & -1 & 1 & -1\\1 & 1 & 0 & 0\\0 & 0 & 1 & 1\end{bmatrix} \to \begin{bmatrix}1 & 1 & 0 & 0\\0 & -1 & 1 & -1\\0 & 0 & 1 & 1\end{bmatrix}$$

P_3 basis $x, x-1, x^2+1$ standard basis $1, x, x^2$

as vectors $B\begin{bmatrix}0 & -1 & 1\\1 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ as vectors $\begin{matrix}1\\0\\0\end{matrix}, \begin{matrix}0\\1\\0\end{matrix}, \begin{matrix}0\\0\\1\end{matrix}$

in standard $3 - 2x + 4x^2 = \begin{bmatrix}3\\-2\\4\end{bmatrix}_E$

$3 - 2x + 4x^2 = \begin{bmatrix} \\ \\ \end{bmatrix}_B$?

use: $\boxed{[v]_E = B[v]_B \text{ or } [v]_B = B^{-1}[v]_E}$

$$[p]_B = \begin{bmatrix}0 & -1 & 1\\1 & 1 & 0\\0 & 0 & 1\end{bmatrix}^{-1}\begin{bmatrix}3\\-2\\4\end{bmatrix}_E$$

$$\left[\begin{array}{ccc|c} 0 & -1 & 1 & 3 \\ 1 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 1 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

$$P = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}_B$$

$$3 - 2x + 4x^2 = -3(x) + 1(x-1) + 4(x^2+1)$$

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15 $7(x) - 12(x-1) + 13(x^2+1) =$ standard?

$a + bx + cx^2$

$$\begin{bmatrix} 0 & -1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 7 \\ -12 \\ 13 \end{bmatrix}_B = \begin{bmatrix} 25 \\ -5 \\ 13 \end{bmatrix}_E$$

$$= 25 - 5x + 13x^2$$

3.6 given $A \xrightarrow[\text{ops}]{\text{row}} U$ in gauss, gauss/jordan

$$\begin{bmatrix} 1 & -2 & 3 & -2 & 1 \\ 0 & 0 & 1 & -1 & 2 \\ 2 & -4 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$a_{12} = -2a_{11}$

$a_{14} = a_{11} - a_{13}$

$$A = \begin{bmatrix} 1 & -2 & 3 & -2 & 1 \\ 0 & 0 & 1 & -1 & 2 \\ 2 & -4 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 & -2 & 1 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & -5 & 5 & 1 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 3 & -2 & 1 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 & -2 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = U$$

Facts: #1 ind. col's a_1, a_3, a_5

dep col's with dep. eqn's

$$a_2 = -2a_1$$

$$a_4 = a_1 - a_3$$

#2 $\text{rank}(A) = 3$ (b/c # of lead's)

#3 $N(A) \to$ solve $A\mathbf{x} = 0$ what is nice? $A\mathbf{x} = 0 \Rightarrow U\mathbf{x} = 0$

$$[A \mid 0] \to \left[\begin{array}{ccccc|c} 1 & -2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{array}{l} x_1 = 2\alpha - \beta \\ x_3 = \beta \\ x_5 = 0 \end{array}$$

$x_2 = \alpha \quad x_4 = \beta$

$$A\mathbf{x} = 0 \Rightarrow \mathbf{x} = \begin{bmatrix} 2\alpha - \beta \\ \alpha \\ \beta \\ \beta \\ 0 \end{bmatrix} = \alpha \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$N(A) = \text{Span}\left(\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right)$$

$\dim(N(A)) = 2$ (nullity of A)

#4
$$\begin{bmatrix} 1 & -2 & 3 & -2 & 1 \\ 0 & 0 & 1 & -1 & 2 \\ 2 & -4 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{col space of } A = \text{Span}\left(\begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \right)$$

$$\text{row space of } A = \text{Span}\left([1\ -2\ 0\ 1\ 0], [0\ 0\ 1\ -1\ 0], [0\ 0\ 0\ 0\ 1] \right)$$