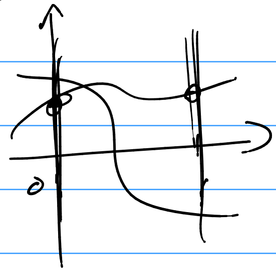
Math 511

Q's 3.3 #9 $C[0,1]$ all cont. functions over $[0,1]$



9d) $f_1 = e^x$, $f_2 = e^{-x}$, $f_3 = e^{2x}$

ind. check?

$c_1 v_1 + c_2 v_2 + \dots + c_k v_k = 0$

$$c_1 e^x + c_2 e^{-x} + c_3 e^{2x} = 0$$

try Wronskian

$$\begin{vmatrix} e^x & e^{-x} & e^{2x} \\ e^x & -e^{-x} & 2e^{2x} \\ e^x & e^{-x} & 4e^{2x} \end{vmatrix} = \begin{vmatrix} e^x & e^{-x} & e^{2x} \\ 0 & 2e^{-x} & -e^{2x} \\ 0 & 0 & -3e^{2x} \end{vmatrix}$$

$$= (e^x)(2e^{-x})(-3e^{2x}) = -6e^{2x}$$

$W_k(x) = -6e^{2x}$ we ask is $W(x) \neq 0$ for some x between $0,1$?

Yes! $\boxed{\text{Independent}}$

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tech #2 $f_1 = e^x$, $f_2 = e^{-x}$, $f_3 = e^{2x}$ over $[0,1]$

check ind? Solve $c_1 e^x + c_2 e^{-x} + c_3 e^{2x} = 0$

pick 3 sample x's

$x=0$ $\quad c_1 + c_2 + c_3 = 0$

$x = 1/2$ $\quad \sqrt{e}\, c_1 + \frac{1}{\sqrt{e}} c_2 + e c_3 = 0$

$x = 1$ $\quad e c_1 + \frac{1}{e} c_2 + e^2 c_3 = 0$

use det $\begin{vmatrix} 1 & 1 & 1 \\ \sqrt{e} & \frac{1}{\sqrt{e}} & e \\ e & \frac{1}{e} & e^2 \end{vmatrix} = ?$

$$= \begin{vmatrix} \frac{1}{\sqrt{e}} & e \\ \frac{1}{e} & e^2 \end{vmatrix} - \begin{vmatrix} \sqrt{e} & e \\ e & e^2 \end{vmatrix} + \begin{vmatrix} \sqrt{e} & \frac{1}{\sqrt{e}} \\ e & \frac{1}{e} \end{vmatrix}$$

$$= (e^{3/2} - 1) - (e^{5/2} - e^2) + (e^{-1/2} - e^{1/2})$$

$\neq 0$ so $\boxed{\text{ind}}$

3.5 #9 $B = [x, 1]$ ordered bases $D = [2x-1, 2x+1]$

$\boxed{P_2 \quad p = ax+b}$

vectors: $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}$ $p = \begin{bmatrix} a \\ b \end{bmatrix}$

know: $B[x]_B = D[x]_D$

9b) transition matrix from $[2x-1, 2x+1]$ to $[x, 1]$
D to B

$$[x]_B = \boxed{B^{-1}D}[x]_D$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}}$$

9b) other way $D^{-1}B$

$$\begin{bmatrix} 2 & 2 \\ -1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \boxed{\text{do @ home}}$$

P_3 Basis B is $[1, 1+x, 1+x^2]$ ordered basis

Basis D is $[-2+x, x^2, x-x^2]$

→ P_3 $p(x) = a + bx + cx^2 \longrightarrow \mathbb{P} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

so $B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $D = \begin{bmatrix} -2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

convert from D coord to B coord?

use what $\boxed{\text{everyone}}$ knows $D[x]_D = B[x]_B$

→ $[x]_B = \underbrace{\boxed{B^{-1}D}}_{\text{transition matrix}}[x]_D$

$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -2 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} = ?$

$\boxed{\text{Exam 2}}$ ch 3 only 10 probs @ 10pts

90pts = 100%

Know: $\mathbb{R}^n$, $\mathbb{R}^{m \times n}$, P_n, $C[a,b]$ vector spaces

$\boxed{3.1}$ (1 prob)

- give you the 10 axioms

(1) is V with $x + y$, αx given a vector space.

Check $\boxed{\text{all}}$ axioms.

$\boxed{3.2}$ (2 probs)

① ② } given S a subset of a V
is it a subspace?

$\boxed{3.3}$ (2 probs)

① ② } given $v_1, v_2, \ldots, v_k$ of V
are they lin. ind? dep?

$\boxed{3.4}$ (2 prob) basis, spans, of $\mathbb{R}^n$, P_n

① dim of Span$(v_1, v_2, \ldots, v_k)$
→ give basis of the span.

② Make a non-standard basis for ...

ex P_3 standard: $\{1, x, x^2\}$

non-standard: $\{1-x, 3x^2, 2-x^2\}$

$\boxed{3.5}$ (2 prob)

① Translation to / from standard coordinates
non-standard coordinate.

② translation to/from two non-standard basis.

$\boxed{3.6}$ (1 prob) given $\underline{A} \xrightarrow[\text{ops}]{\text{row}} U$ upper triangular gauss-jordan

__State:__ (1) rank, nullity, dep equ's of A

(2) col space of A = span(   )

(3) row space of A = span(   )

(4) $N(A)$ = span(   )