Math 511

Q5 4.1 #6 $L: \mathbb{R}^2 \to \mathbb{R}^3$ $\qquad L\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = \begin{bmatrix} \\ \\ \end{bmatrix}$ ← in $\mathbb{R}^3$

6b) $L\left(\begin{bmatrix} \square \\ \bigcirc \end{bmatrix}\right) = \begin{bmatrix} \square \\ \bigcirc \\ \square + 2\bigcirc \end{bmatrix}$

check: given X, Y in $\mathbb{R}^2$ $\qquad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$

check: $L(\alpha X + \beta Y) \overset{?}{=} \alpha L(X) + \beta L(Y)$

left: linear combo $\alpha X + \beta Y = \begin{bmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \end{bmatrix}$

transform $L(\alpha X + \beta Y) = \begin{bmatrix} \alpha x_1 + \beta y_1 \\ \alpha x_2 + \beta y_2 \\ (\alpha x_1 + \beta y_1) + 2(\alpha x_2 + \beta y_2) \end{bmatrix}$

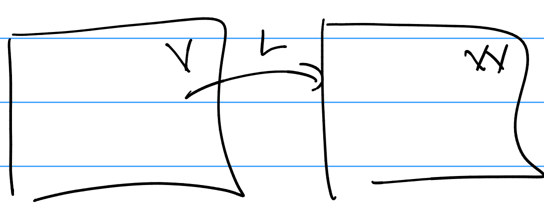
Same!

right $\alpha L(X) + \beta L(Y)$

$= \alpha \begin{bmatrix} x_1 \\ x_2 \\ x_1 + 2x_2 \end{bmatrix} + \beta \begin{bmatrix} y_1 \\ y_2 \\ y_1 + 2y_2 \end{bmatrix} = \begin{bmatrix} \text{simplify} \end{bmatrix}$

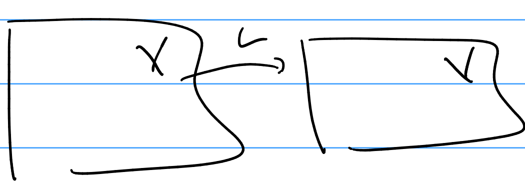
4.1 #8 $\quad C$ is $n \times n$ $\qquad$ L is a linear transform from V to W

Names: a)



$L: V \to W$

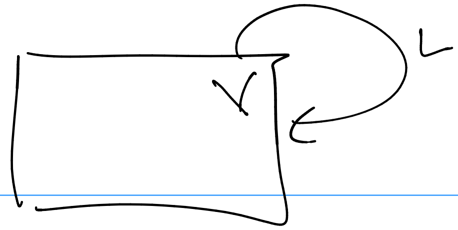
b)



$L: V \to V$

L is a linear transform from V to V

4



call L a linear operator on V.

so #8 is $L(A) = B$ a linear operator

A: matrix in $\mathbb{R}^{n \times n}$ — B: matrix in $\mathbb{R}^{n \times n}$

(8a)

what does L do to A? $L(A) = CA + AC$

is this a linear operator? ← linear transform

check for linear transform?

$$L(\alpha A + \beta B) \overset{?}{=} \alpha L(A) + \beta L(B)$$

left side: #1 linear combo: $\alpha A + \beta B = [\alpha a_{ij} + \beta b_{ij}]$

#2 transform: $C[\alpha a_{ij} + \beta b_{ij}] + [\alpha a_{ij} + \beta b_{ij}]C$

$$[c_{ij}][d_{ij}]$$

$$[\vec{c}_i \, d_j]$$

right side: similar

Note: for linear transform test..

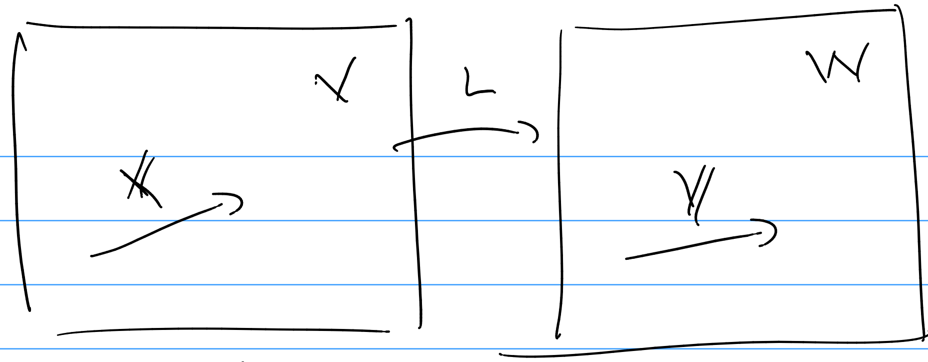
test #1: $\boxed{L(\alpha_1 V_1 + \alpha_2 V_2) \overset{?}{=} \alpha_1 L(V_1) + \alpha_2 L(V_2)}$ ✓

test #2:

check addition: $L(V_1 + V_2) \overset{?}{=} L(V_1) + L(V_2)$

check scalar mult: $L(\alpha V_1) \overset{?}{=} \alpha L(V_1)$ ✓

4.2/4.3



Vector space V is $\dim(V)=n$, basis $\{\mathbb{b}_1, \mathbb{b}_2, \ldots, \mathbb{b}_n\} = B$

Vector space W is $\dim(W)=m$, basis $\{\mathbb{d}_1, \mathbb{d}_2, \ldots, \mathbb{d}_n\} = D$

$[\mathbb{x}]_E$ coord of $\mathbb{x}$ using standard

$[\mathbb{x}]_B$ coord of $\mathbb{x}$ using basis B

$[\mathbb{y}]_E$ standard coord.

$[\mathbb{y}]_D$ coord. using basis D

and $[\mathbb{x}]_E = B[\mathbb{x}]_B$

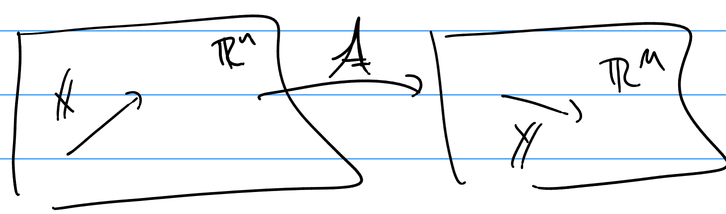
$[\mathbb{y}]_E = D[\mathbb{y}]_D$

$[\mathbb{x}]_B = B^{-1}[\mathbb{x}]_E$

$[\mathbb{y}]_D = D^{-1}[\mathbb{y}]_E$

know: $A\mathbb{x} = \mathbb{y}$

A is $m \times n$



is a linear transformation from $\mathbb{R}^n$ to $\mathbb{R}^m$

Q: given $L : \mathbb{R}^n \to \mathbb{R}^m$ can I find a matrix so that

$L(\mathbb{x}) = \mathbb{y}$ if and only if $A\mathbb{x} = \mathbb{y}$

ex $L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a+b \\ b+c \end{bmatrix}$ is $L : \mathbb{R}^3 \to \mathbb{R}^2$

do this @ home.

vs

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a+b \\ b+c \end{bmatrix}$$

$$L(X) = L([X]_E) = L(x_1 e_1 + x_2 e_2 + \cdots + x_n e_n)$$

$$= x_1 L(e_1) + x_2 L(e_2) + \cdots + x_n L(e_n)$$

$$= \begin{bmatrix} L(e_1) & L(e_2) & \cdots & L(e_n) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$$= \underbrace{\boxed{\begin{bmatrix} L(e_1) & L(e_2) & \cdots & L(e_n) \end{bmatrix}}}_{m \times n \text{ matrix}} [X]_E$$

thm: $L : \mathbb{R}^n \to \mathbb{R}^m$ then let $A = \begin{bmatrix} L(e_1) & L(e_2) & \cdots & L(e_n) \end{bmatrix}$

<u>and</u> $L(X) = A[X]_E$

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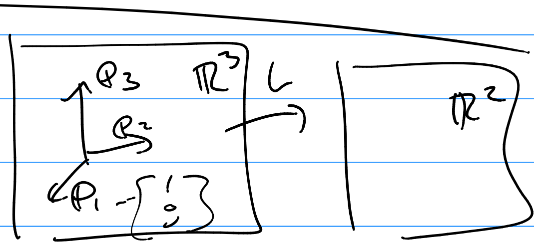
ex: $L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a+b \\ b+c \end{bmatrix}$



$$A = \begin{bmatrix} L\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right) & L\left(\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right) & L\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right) \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$L : V \to W$ $\quad$ $\dim(V) = n$ $\quad$ V's standard coord are $e_1\ e_2 \cdots e_n$

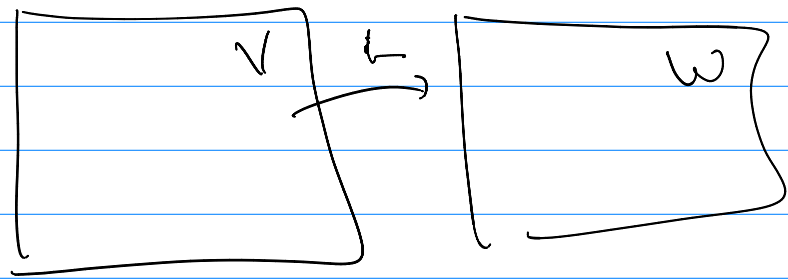
$$A = \begin{bmatrix} L(e_1) & L(e_2) & & L(e_n) \end{bmatrix}$$

call A the <u>standard</u> representation of L.

So $\quad L(x) = y \qquad\qquad A[x]_E = [y]_E$

but... what if we also have non-standard coord?

$\dim(V) = n$ $\qquad\qquad\qquad\qquad$ $\dim(W) = m$



basis $\quad B = [b_1 \dots b_n] \qquad\qquad D = [d_1 \dots d_m]$

#1 $\quad L : V \to W$ has standard rep $\underset{A}{A}[x]_E = [y]_E$

#2 $\quad$ what about L as a matrix $(= S)$

so that $\qquad S[x]_B = [y]_D$

$$D^{-1}\underbrace{A\underbrace{B[x]_B}_{[x]_E}}_{[y]_E} = [y]_D$$

$$S = D^{-1}AB$$ matrix rep. for L from coord in B to coord in D.