Math 511

Q's 4.1 (#17a) $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix}$

Ker (L) see $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ So $\mathrm{Ker}(L) = \left\{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\right\}$

$$\begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

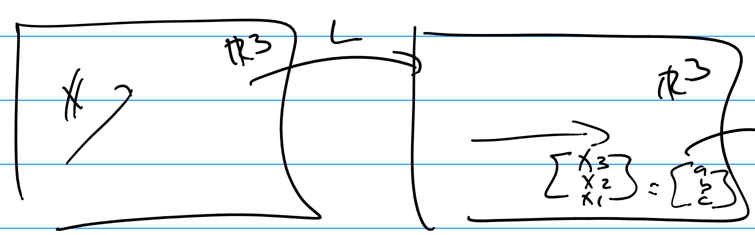
17b $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}$

for Ker(L) $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ $x_1 = 0$, $x_2 = 0$, $x_3 = \alpha$

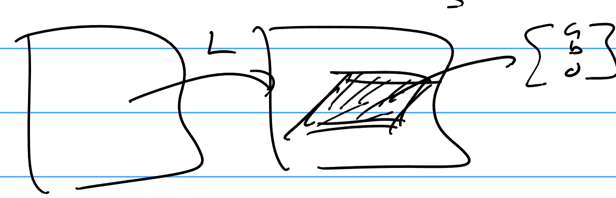
$$\mathrm{Ker}(L) = \mathrm{Span}\left(\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right)$$

Range? 17a) $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_3 \\ x_2 \\ x_1 \end{bmatrix} \overset{?}{=}$ anywhere? in $\mathbb{R}^3$



Range $L = \mathbb{R}^3$

17b $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}$

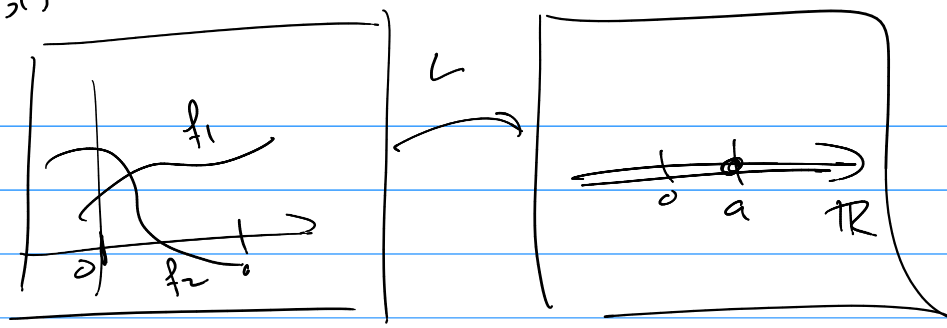


Range of L xy plane

$\mathbb{R}^2$ of $z = 0$

$C[0,1]$

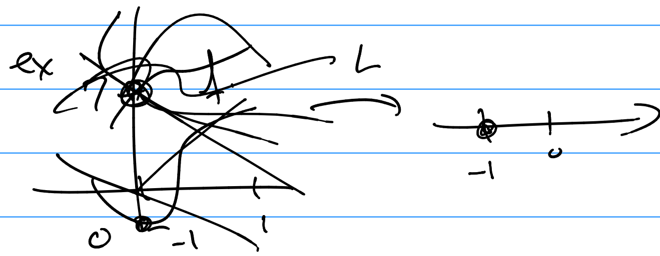
(1)

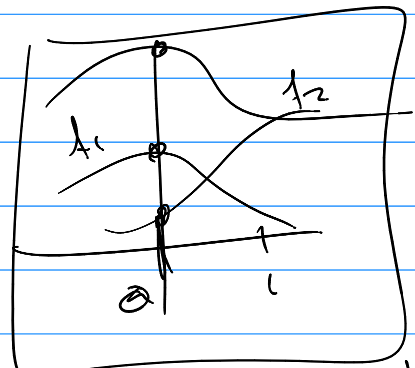


$L(f_1) = a$

1a) $L(f(x)) = f(0)$

ex





$$\boxed{L(\alpha f_1 + \beta f_2) \overset{?}{=} \alpha L(f_1) + \beta L(f_2)}$$

Know: $(\alpha f_1 + \beta f_2)(x) = \alpha f_1(x) + \beta f_2(x)$

left. $L((\alpha f_1 + \beta f_2)(x)) = (\alpha f_1 + \beta f_2)(0)$

$= \alpha f_1(0) + \beta f_2(0)$

Same!

vs

right. $\alpha L(f_1(x)) + \beta L(f_2(x)) = \alpha f_1(0) + \beta f_2(0)$

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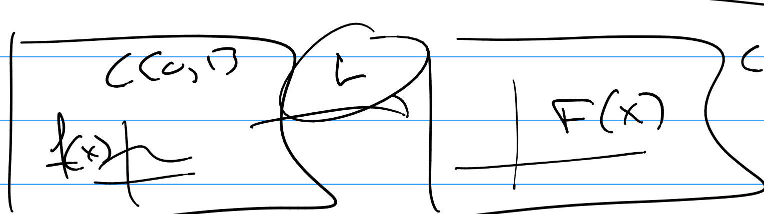
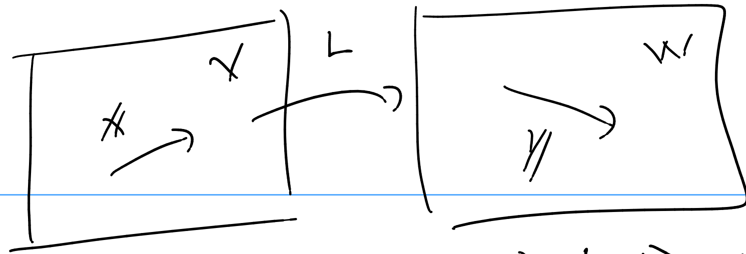
ex #(1)



$$F(x) = \int_0^x f(t)\,dt$$

4.3



$\dim(V) = n$ $\qquad$ $\dim(W) = m$

Basis $B = \{b_1, b_2 \ldots b_n\}$ $\qquad$ Basis $D = \{d_1, d_2 \ldots d_m\}$

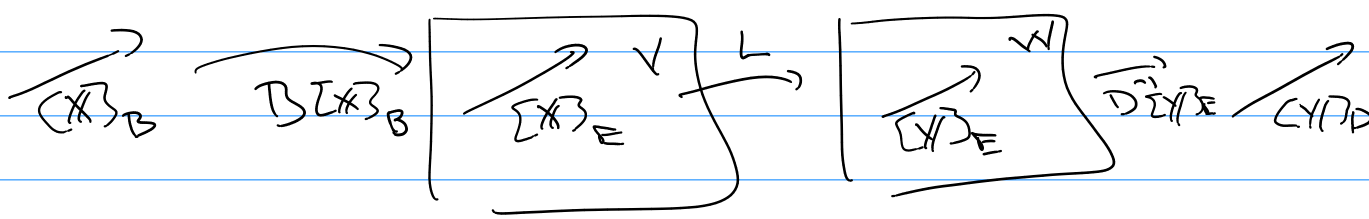
Given $L(x) = y$

#1 Standard matrix form of L is $A[x]_E = [y]_E$

$$A = [\, L(e_1) \;\; L(e_2) \ldots L(e_n) \,]$$

#2 want to use basis B of V vector space
basis D of W vector space

$$D^{-1}AB[x]_B = [y]_D$$



$D^{-1}AB$ is the matrix representing L from Basis B to Basis D.

Note: Calculating $D^{-1}AB$

#1 $AB = A[b_1 \;\; b_2 \ldots b_n]$
$= [Ab_1 \;\; Ab_2 \ldots Ab_n]$

$= [L(b_1) \;\; L(b_2) \ldots L(b_n)]$

a) we can use Matrix for L in standard to standard

$$A = [L(e_1)\ L(e_2) \cdots L(e_n)]$$ ✓

y) matrix for L in B to standard

$$AB = [L(b_1)\ L(b_2) \cdots L(b_n)]$$

#2 $D^{-1}AB\,[x]_B$ computationally "faster" way ..

$$[\,D \mid L(b_1)\ L(b_2) \cdots L(b_n)\,]$$

rowops

$$[\,I \mid D^{-1}AB\,]$$

Linear Operator $L : V \to V$ we have standard Basis for V we can also have Basis B

#1 L as standard matrix

$$A = [L(e_1)\ L(e_2) \cdots L(e_n)] \qquad A[x]_E = [y]_E$$

#2 L a matrix using Basis B for vectors

$$(B^{-1}AB)[x]_B = [y]_B$$

matrix

call it $T = B^{-1}AB$

or

$$BTB^{-1} = A$$

$M_1 \quad M_2$

$$M_2 = N M_1 N^{-1}$$

Def A, T are similar if a non-singular matrix ~~B~~ N exists (means ~~B^{-1}~~ N also, exists)

and

$$A = B T B^{-1}$$

or

$$T = B^{-1} A B$$

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why? chapter 6

we will have a transformation for standard coord.

$$A = E D E^{-1}$$

diagonal matrix.