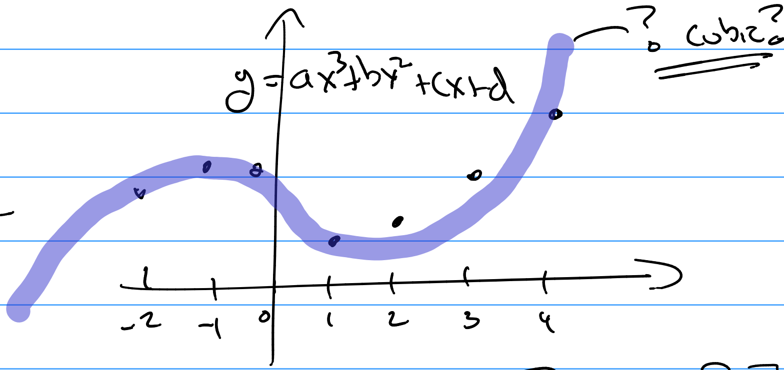
Math 511

$A\mathbb{x} = \mathbb{b}$

overdet. systems (no solution?)

(x,y)	
$(-2,2)$	$(-2)^3a + (-2)^2b + (-2)^1c + d = 2$
$(-1,3)$	$(-1)^3a + (-1)^2b + (-1)c + d = 3$
$(-1/2, 3)$	$(-1/2)^3a + (-1/2)^2b + (-1/2)c + d = 3$
$(1,1)$	$\vdots$
$(2, 3/2)$	
$(3,2)$	
$(4,3)$	$(4)^3a + (4)^2b + 4c + d = 3$



$$\to \begin{bmatrix} (-2)^3 & (-2)^2 & -2 & 1 \\ (-1)^3 & (-1)^2 & -1 & 1 \\ (-1/2)^3 & (-1/2)^2 & -1/2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ (4)^3 & (4)^2 & 4 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 3 \\ \vdots \\ 3 \end{bmatrix}$$

$7\times4 \qquad 4\times1 \qquad 7\times1$

try gauss-jordan $\to$ no soln!

$$A \ \mathbb{x} = \mathbb{b}$$

Solve $\boxed{A^T A}\,\mathbb{x} = A^T\mathbb{b}$ can be solved.

$4\times4 \quad 4\times1 \quad 4\times1$

4.3 #2 $\quad \mathbb{R}^2 \quad U = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \quad V = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$

$L([\mathbb{x}]_E) = \begin{bmatrix} -x_1 \\ x_2 \end{bmatrix}_E \qquad A = \left[L\begin{pmatrix}1\\0\end{pmatrix} \ L\begin{pmatrix}0\\1\end{pmatrix} \right]$

L in standard matrix

linear trans.

$A = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

a) Matrix from $[\mathbb{x}]_U$ to $[\mathbb{x}]_V$

$[\mathbb{y}]_V = V^{-1}AU[\mathbb{x}]_U$

a) change of basis $V^{-1}U$

$$S = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

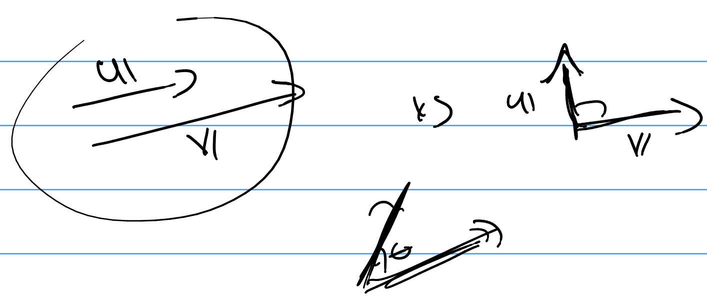
$$= \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix} =$$

$$\left[\begin{array}{cc|cc} 2 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{array}\right] = \left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 2 & 1 & -1 & 1 \end{array}\right]$$

$$= \left[\begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & -3 & -1 \end{array}\right]$$

5.4 orthonal? - what can we do for $\boxed{\text{any}}$ vector space V?

→ contribution



for any vector space is there something like...

→ $X^T Y = 0$ means $X \perp Y$ for $\mathbb{R}^n$

$$x_1y_1 + x_2y_2 + \dots + x_ny_n = 0$$

For other vector spaces we need a binary operator on X, Y that takes X, Y and gives a scalar.

Notation: $\langle X, Y \rangle$ = scalar and #1 $\langle X, X\rangle \geq 0$ and only $\langle 0, 0\rangle = 0$

#2 $\langle X, Y\rangle = \langle Y, X\rangle$

#3 $\langle \alpha X + \beta Y, Z\rangle = \alpha\langle X, Z\rangle + \beta\langle Y, Z\rangle$

Def: If you have V, a vector space, and create a $\langle X, Y\rangle$ that has the above properties

① call $\langle X, Y\rangle$ an inner product

② call V with $\langle X, Y\rangle$ an inner prod. space

Facts: #1 $\langle X, Y\rangle = 0$ then $X \perp Y$ (orthogonal)

#2 (length) $\|X\|^2 = \langle X, X\rangle$

norm or magnitude

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ex: $\mathbb{R}^n$ ① $\langle X, Y\rangle = X^T Y$ is an inner product.

② $\langle X, Y\rangle = w_1 x_1 y_1 + w_2 x_2 y_2 + \cdots + w_n x_n y_n$ all $w_i > 0$

$\langle X, X\rangle \geq 0$ and only $\langle 0, 0\rangle = 0$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

$\mathbb{R}^{n\times n}$ $\quad A=[a_{ij}] \quad B=[b_{ij}]$

(1) $$\langle A,B\rangle = \begin{pmatrix} a_{11}b_{11}+a_{12}b_{12}+\dots+a_{1n}b_{1n} \\ +a_{21}b_{21}+a_{22}b_{22}+\dots+a_{2n}b_{2n} \\ + \dots \\ + \dots + a_{nn}b_{nn} \end{pmatrix} = \sum\sum a_{ij}b_{ij}$$

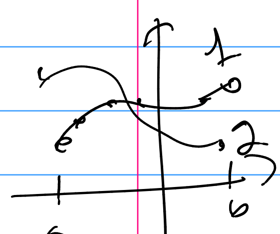
(2) $$\underline{\underline{\langle A,B\rangle = \sum\sum w_{ij}a_{ij}b_{ij}}} \quad \text{with } w_{ij}>0$$

ex $$\left\| \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \right\| = \left(\left\langle \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \end{bmatrix} \right\rangle \right)^{1/2}$$

$$= \sqrt{1^2+2^2+3^2+(-1)^2+(0)^2+(1)^2}$$

$$= \sqrt{16} = \boxed{4}$$

$C[a,b]$ (1) $\langle f,g\rangle = \int_a^b f(x)g(x)\,dx$



(2) $\langle f,g\rangle = \int_a^b w(x)f(x)g(x)\,dx \quad w(x)>0$
