Math 511

$\boxed{\text{Q's}}$ $\det(A - \lambda I) = 0$

$A = \begin{bmatrix} 3 & 4 \\ 3 & 2 \end{bmatrix}$ ① $[A - \lambda I] = \begin{bmatrix} 3-\lambda & 4 \\ 3 & 2-\lambda \end{bmatrix}$

② λ_i? $\det\begin{pmatrix} 3-\lambda & 4 \\ 3 & 2-\lambda \end{pmatrix} = 0$

$(3-\lambda)(2-\lambda) - 12 = 0$
$6 - 5\lambda + \lambda^2 - 12 = 0 \rightarrow \lambda^2 - 5\lambda - 6 = 0$
$(\lambda - 6)(\lambda + 1) = 0$
$\lambda = 6 \quad \lambda = -1$

③ for each λ_i $X_i = ?$

a) $\lambda = 6$ $\left[\begin{array}{cc|c} -3 & 4 & 0 \\ 3 & -4 & 0 \end{array}\right] \rightarrow X = ?$

b) $\lambda = -1$ $\left[\begin{array}{cc|c} 4 & 4 & 0 \\ 3 & 3 & 0 \end{array}\right] \rightarrow X = ?$

6.2 1d $y_1' = y_1 - y_2$ $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$
$y_2' = y_1 + y_2$

① $[A - \lambda I] = \begin{bmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix}$

② $\det\left(\begin{bmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{bmatrix}\right) = 0 \rightarrow (1-\lambda)^2 + 1 = 0$
$\rightarrow (1-\lambda)^2 = -1 \rightarrow (1-\lambda) = \pm i$
$\lambda = 1 \pm i \quad \lambda_1 = 1+i \quad \lambda_2 = 1-i$

③ a) for $\lambda = 1+i$ $\left[\begin{array}{cc|c} 1-(1+i) & -1 & 0 \\ 1 & 1-(1+i) & 0 \end{array}\right]$

$$\begin{bmatrix} -i & -1 & | & 0 \\ 1 & -i & | & 0 \end{bmatrix} \xrightarrow{r_1(i) = \text{New } r_1} \begin{bmatrix} 1 & -i & | & 0 \\ 1 & -i & | & 0 \end{bmatrix} \to \begin{bmatrix} 1 & -i & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$$

x_2 is free

$x_2 = \alpha$

$x_1 - i x_2 = 0$

$x_1 = i\alpha = \alpha i$

$$X = \begin{bmatrix} \alpha i \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} i \\ 1 \end{bmatrix}$$

b) $\lambda_2 = 1 - i \quad X_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

So $\lambda_1 = 1+i \quad X_1 = \begin{bmatrix} i \\ 1 \end{bmatrix} \qquad \lambda_2 = 1-i \quad X_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

Soln $\quad Y = c_1 e^{\lambda_1 t} X_1 + c_2 e^{\lambda_2 t} X_2$

$$Y = c_1 e^{(1+i)t} \begin{bmatrix} i \\ 1 \end{bmatrix} + c_2 e^{(1-i)t} \begin{bmatrix} -i \\ 1 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} c_1 i e^{(1+i)t} + c_2(-i) e^{(1-i)t} \\ c_1 e^{(1+i)t} + c_2 e^{(1-i)t} \end{bmatrix}}$$

Exam 4

Exam 1 → "take" 2 probs

Exam 2 → "take" 2 probs

Exam 3 → "take" 2 probs

Ch 6   4 probs   (1) λ, X triangular matrix

(2) λ, X normal matrix

(3) $AX = XD$

$A = XDX^{-1}$

$D = X^{-1}AX$

(4) Diff Eq system

Ch6 6.3 diagonalization

If A has $\lambda_1, \lambda_2, \ldots, \lambda_n$ eigen values
with $x_1, x_2, \ldots, x_n$ eigen vectors

call $X = [x_1 \ x_2 \ \cdots \ x_n]$
$D = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & \cdots & \lambda_n \end{bmatrix}$

X and D non-uniq.

ex $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = A$ $\quad \lambda_1 = 1+i \quad x_1 = \begin{bmatrix} i \\ 1 \end{bmatrix}$
$\lambda_2 = 1-i \quad x_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

$D = \begin{bmatrix} 1-i & 0 \\ 0 & 1+i \end{bmatrix} \quad X = \begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix}$

$$A[x_1 \ x_2 \cdots x_n] = [Ax_1 \ Ax_2 \cdots Ax_n]$$
$$= [\lambda_1 x_1 \ \lambda_2 x_2 \quad \lambda_n x_n]$$
$$= [x_1 \ x_2 \cdots x_n] \begin{bmatrix} \lambda_1 & & 0 \\ 0 & \ddots & \lambda_n \end{bmatrix}$$

$$\boxed{AX = XD}$$

Now if X^{-1} exists $\qquad X = [x_1 \ x_2 \cdots x_n]$

x_i must be linearly independent

If X is invertible $\qquad AX = XD$

$$A = XDX^{-1} \qquad \boxed{D} = X^{-1}\boxed{A}X \quad [v]_X$$

Consider (if x_i are lin. ind.)

x_i are a basis

trans. in basis of x_i

transform in standard basis

$$D = \begin{bmatrix} \lambda_1 & & 0 \\ & \lambda_2 & \\ 0 & & \ddots \ \lambda_n \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 \\ 0 & .2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}_X = \begin{bmatrix} 1a \\ .2b \end{bmatrix}_X$$