Math 511

$\boxed{Q's}$ 6.2 (26) using e^A, $A = XDX^{-1}$

$$A = [\underline{x_1 \ x_2 \cdots x_n}] \begin{bmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{bmatrix} [x_1 \ x_2 \cdots x_n]^{-1}$$

2 useful facts

① $A^k = [x_1 \ x_2 \cdots x_n] \begin{bmatrix} \lambda_1^k & & 0 \\ & \ddots & \\ 0 & & \lambda_n^k \end{bmatrix} [x_1 \ x_2 \cdots x_n]^{-1}$

② $\exp(A) = e^A = [x_1 \ x_2 \cdots x_n] \begin{bmatrix} e^{\lambda_1} & & 0 \\ & \ddots & \\ 0 & & e^{\lambda_n} \end{bmatrix} [x_1 \ x_2 \cdots x_n]^{-1}$

Now...

$$\begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_n' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ \vdots & \ddots & & \\ \vdots & & & \\ a_{n1} & & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$

Initial values $t=0$

$$\begin{bmatrix} y_1(0) \\ y_2(0) \\ \vdots \\ y_n(0) \end{bmatrix}$$

Soln

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = e^{tA} \begin{bmatrix} y_1(0) \\ \vdots \\ y_n(0) \end{bmatrix}$$

$\to$ if $A = XDX^{-1}$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = X \begin{bmatrix} e^{t\lambda_1} & & & 0 \\ & e^{t\lambda_2} & & \\ 0 & & \ddots & \\ & & & e^{t\lambda_n} \end{bmatrix} X^{-1} \begin{bmatrix} y_1(0) \\ \vdots \\ y_n(0) \end{bmatrix}$$

6.2 (26) $\quad y_1' = y_1 - 2y_2 \qquad Y_0 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

$\quad\quad\quad\;\; y_2' = 2y_1 + y_2$

$A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \qquad \lambda ? \quad X ?$

<u>step 1</u> $\quad \begin{bmatrix} 1-\lambda & -2 \\ 2 & 1-\lambda \end{bmatrix}$

<u>step 2</u> $\quad \det\begin{pmatrix} 1-\lambda & -2 \\ 2 & 1-\lambda \end{pmatrix} = 0 \Rightarrow (1-\lambda)^2 + 4 = 0$

$(1-\lambda)^2 = -4$

$(1-\lambda) = \pm 2i$

$\lambda_1 = 1+2i \qquad \lambda_2 = 1-2i$

<u>Step 3</u> a) for $\lambda_1 = 1+2i$

$$\left[\begin{array}{cc|c} 1-(1+2i) & -2 & 0 \\ 2 & 1-(1+2i) & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} -2i & -2 & 0 \\ 2 & -2i & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 2 & -2i & 0 \\ 2 & -2i & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cc|c} 1 & -i & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{l} x_2 = \alpha \\ x_1 = \alpha i \end{array} \qquad X = \begin{bmatrix} \alpha i \\ \alpha \end{bmatrix} = \alpha \begin{bmatrix} i \\ 1 \end{bmatrix}$$

$\uparrow$ free!

$\lambda_1 = 1+2i \qquad X_1 = \begin{bmatrix} i \\ 1 \end{bmatrix}$

b) $\lambda_2 = 1-2i \qquad X_2 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

Soln

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{(1+2i)t} & 0 \\ 0 & e^{(1-2i)t} \end{bmatrix} \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$= ?$ ← for fun @ home

Exam 4 10 probs @ 10 pts

90 pts = 100%

Exam 1 2 probs from this (don't worry about 7, 9, 11)

?. Solve sys. of eqn's

① sub.

② elim

③ matrices

$x - 2y + 3z = 2$

$2x + y + z = 9$

$-x + y + 0z = -1$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 2 \\ 2 & 1 & 1 & 9 \\ -1 & 1 & 0 & -1 \end{array}\right] \xrightarrow{\text{row ops}} \left[\ \text{Solve} \ \right]$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 1 & 1 \\ -1 & 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 9 \\ -1 \end{bmatrix}$$

no systems word prob.

?. You should be able to do all operations.

$A+B$, αA, Ax, A^{-1}, A^T, etc

?. Know $\det(A)$ and its uses.

$$\det(A) = \begin{vmatrix} 1 & -2 & 3 \\ -1 & 4 & 1 \\ 0 & 3 & 2 \end{vmatrix}$$

two techniques

cofactors

$$\det(A) = -(-1)\begin{vmatrix} -2 & 3 \\ 3 & 2 \end{vmatrix} + (4)\begin{vmatrix} 1 & 3 \\ 0 & 2 \end{vmatrix} - (1)\begin{vmatrix} 1 & -2 \\ 0 & 3 \end{vmatrix}$$

$= \underline{\text{calc}}$

elimination ← row ops

$|A_1| = -|A_2|$ row swap

$|A_1| = |A_2|$ type 3 row op

$|A_1| = \frac{1}{m}|A_2|$

(m) row i = New row i

Exam 2 don't worry about (1, 3, 6)

?. Subspace? check closure?
check $0 \in S$?

?. linear independence?

$\mathbb{R}^n$ ① square? n-vectors of dimension n
use det( )

② not square? a) more vectors than dim. → $\boxed{\text{dep}}$
b) less vectors than dim → check homogeneous sys.

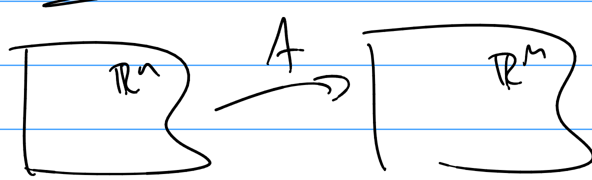
ex $\quad v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \quad v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \quad v_3 = \begin{bmatrix} 0 \\ 2 \\ 4 \\ 5 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 3 & 1 & 4 & 0 \\ 4 & 1 & 5 & 0 \end{array}\right] \rightarrow \text{solns?}$$

? Know the facts of basis, ind., dep, det( ), dimension.

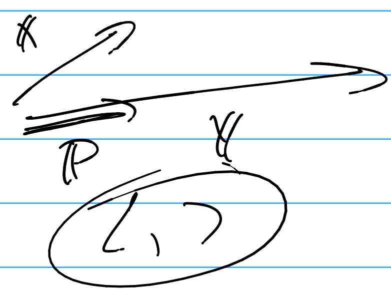
($\mathbb{R}^n$ or P_n)

? change coord?

? 

Exam 3 $\quad$? probs $\quad$ (don't worry about 2, 5, 10)

? $L: V \to W$ is a linear transform?

? 

Projections → scalar, vector, angles

for $\mathbb{R}^n$, $\mathbb{R}^{m \times n}$, P_n inner product spaces

? Can you use coord. of orthonormal basis?

Eigens ch6 4 probs