

Math 322

On Exam?

Soln (class is wee)

Today ch 10 (trees)

Trees:

Undirected Graphs with no
simple circuits is a tree

Since as

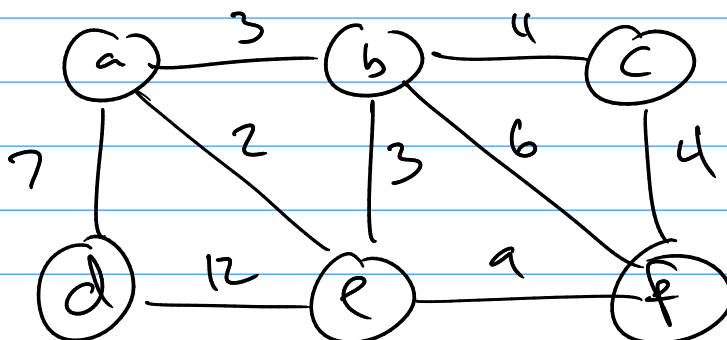
(1) connected and $|E| = |V| - 1$

(2) connected and remove any edge
makes it disconnected

(3) connected and no simple circuits
and add an edge it has
a simple circuit.

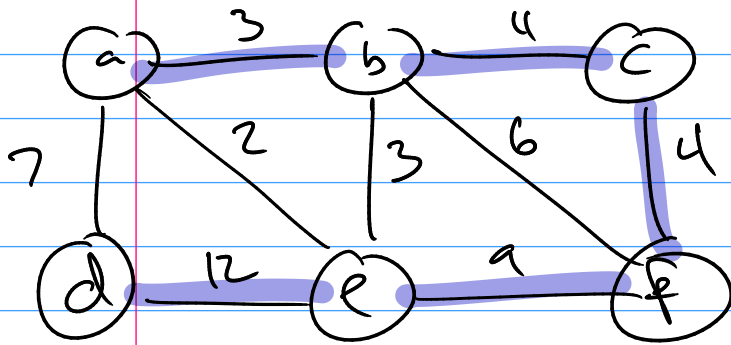
(4) there is a unique simple path between
any two vertices.

Ex given a connected graph that is not a tree.

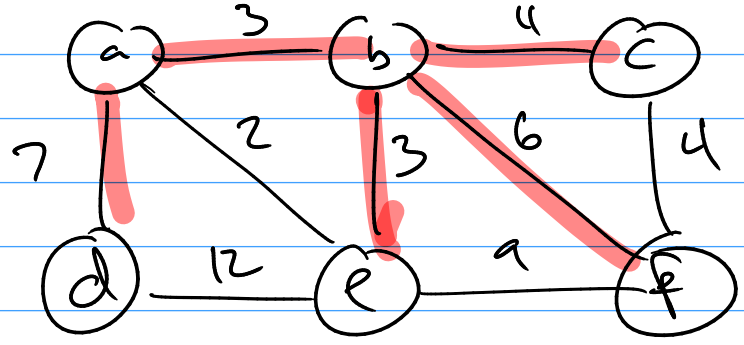


Make a
tree

Def: Spanning tree is a subgraph of a connected undirected graph that is a tree.



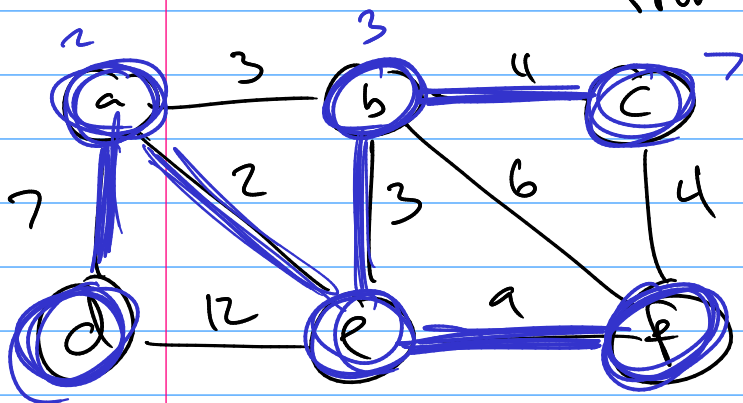
Cost: 32



Cost: 23

to find a spanning tree you need $|V|-1$ edges and it is still connected

① Dijkstra's: find all least cost paths from a specific vertex



Known

e

e, a
 e, b

On Known

a, b, c, d, f

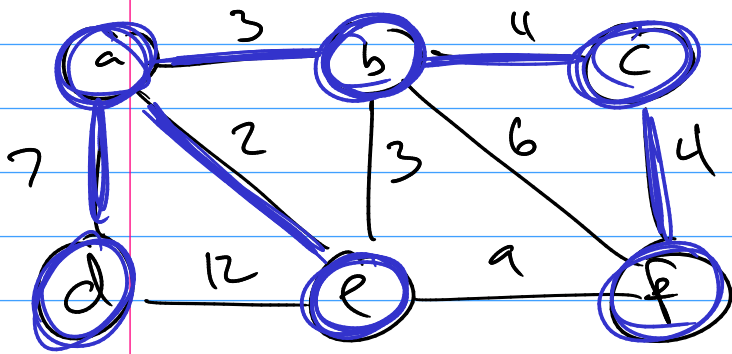
b, c, d, f

c, d, f

Vertex	Path	Cost
e	e	0
a	e, a	2
b	e, b	3
c	e, b, c	7
f	e, f	9
d	e, a, d	9

Cost of tree: 29

Prim's Algorithm : who is cheapest neighbor
(ignore old path costs)



from known to unknown pick cheapest edge.

Known
e

e
e, a, b
e, a, b, c

Unknown
a, b, c, d, f

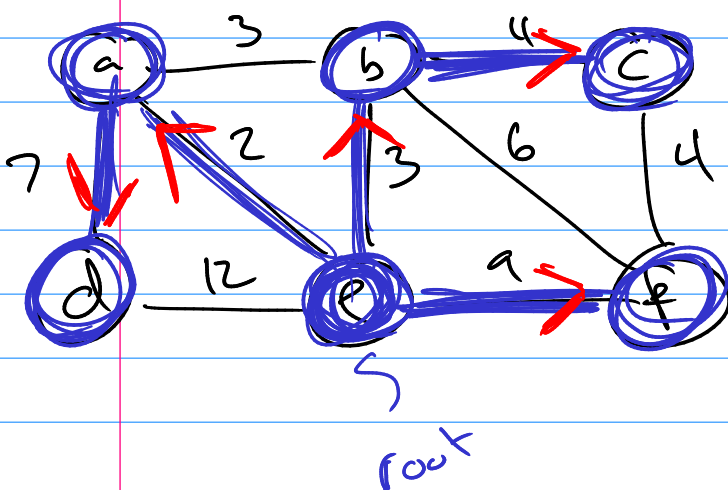
b, c, d, f
c, d, f
a, d

Cost of tree
20

10.3 Rooted trees

fact

there is a unique simple path between any two vertices in a tree.

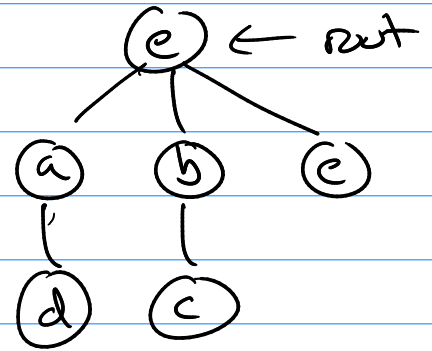
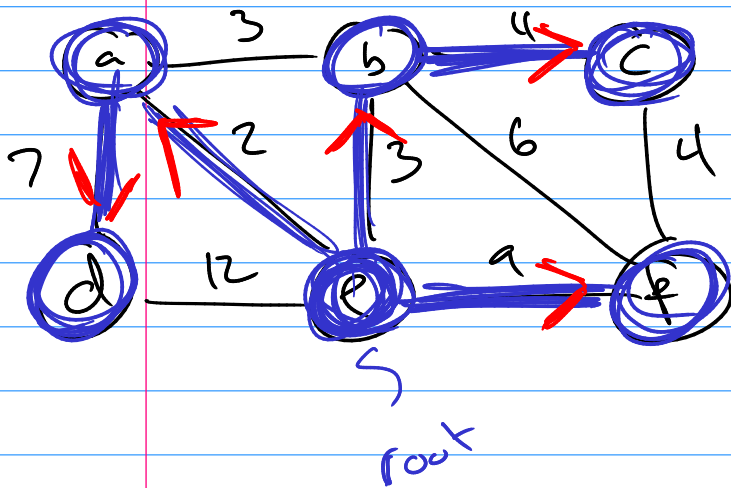


trees are undirected
[but] I can create a direction in a tree by saying direction is "away" from the root

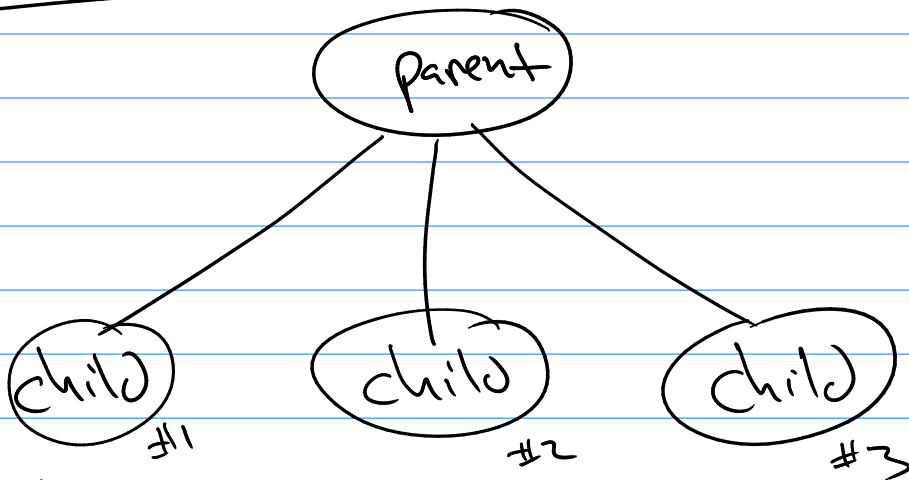
So a rooted tree is a tree where
 I apply direction $\Rightarrow$ away from root

So Rooted tree is a digraph

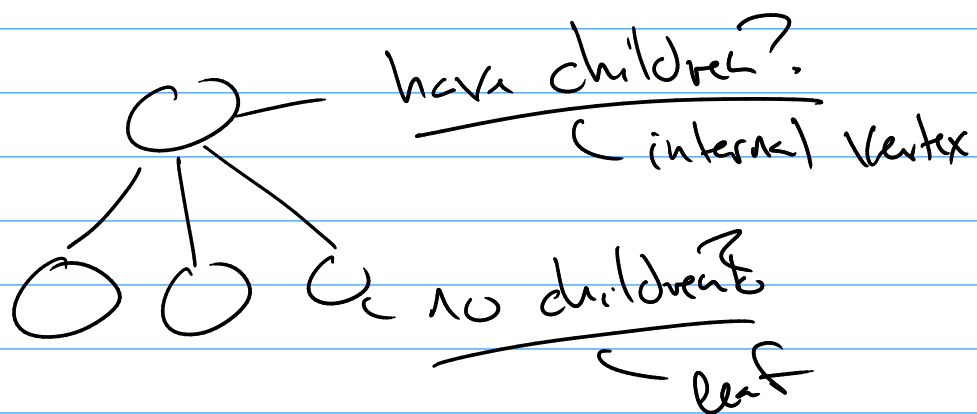
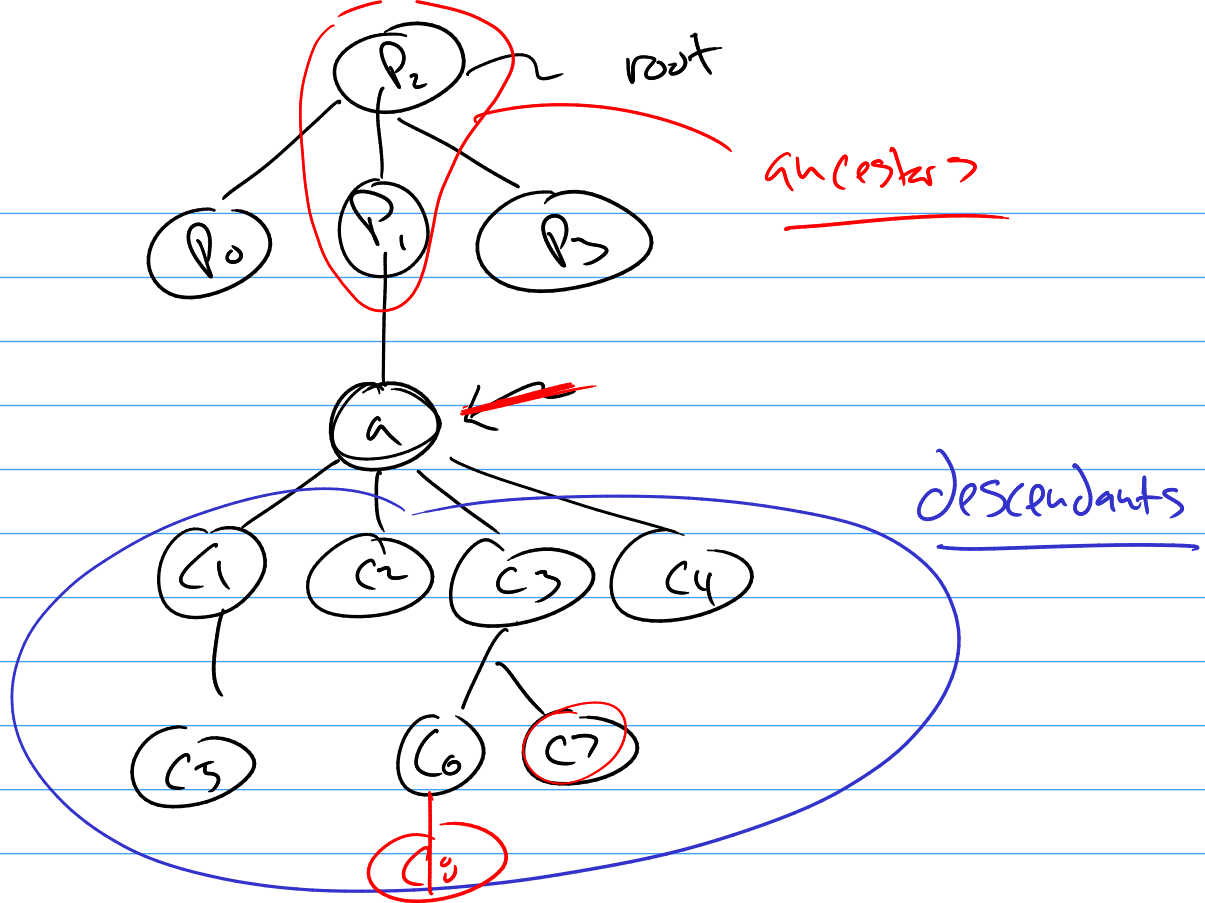
to draw a rooted tree is the opposite of
 a flux diagram.



terms for rooted trees



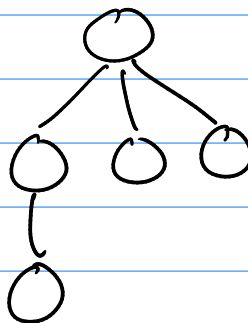
if you order
 children $\equiv$ ordered rooted tree



level = 0

level = 1

level = 2

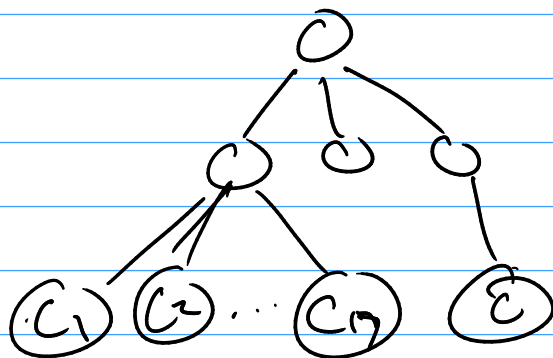


height = max level

level = length of path to root

M-ary tree means each internal vertex has at most M children

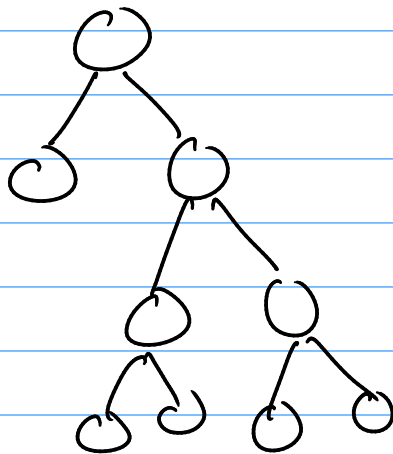
(x)



3-ary tree

Full M-ary tree means each internal vertex has exactly M children

(ex) Full 2-ary (binary)



Full and balanced M-ary tree

all leaves are @ height or height-1

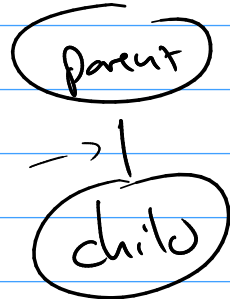
H_n is

① $|V| = i + l$

$i \equiv$ internal vertex
 $l \equiv$ leaf

② $|E| = |V| - 1$

v_c



③ $|children| = |E|$

④ only root has no parent

n is a full map tree $|children| = m \cdot i$

⑤ $|V| = m \cdot i + 1$

So

we have the following system

$$\begin{cases} n = m \cdot i + 1 \\ n = i + l \end{cases}$$

or
to find
other

find n, i, l ?

only need one

2

- $|V| = n$
- full m -ary tree
- $i \equiv$ internal
- $l \equiv$ leaf