

# Math 344

10.1 (1c, 3)  
10.2 (4ac)

Remember to do all examples  
like normal

Trees

Our last thm for trees

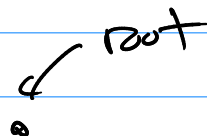
HL

$$l \leq M^h$$

Most number of leaves for tree is  $M^h$

PF

(induct)

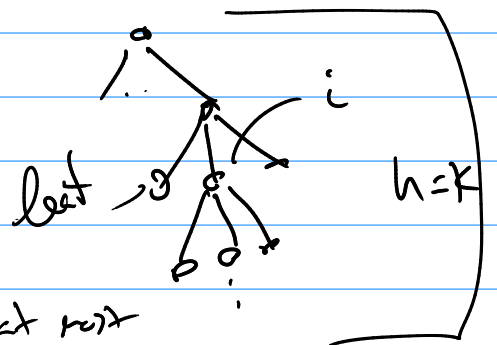
Base:  $h=0 \rightarrow$  tree:   $l=1$ ,  $M^0=1$   
 $1 \leq 1$  is true

Inductive:

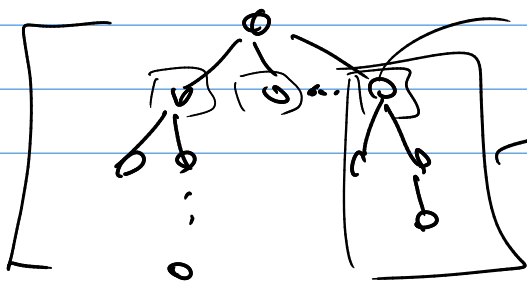
assume for tree of height  $k$

$$l \leq M^k$$

Show Most leaves for height  $k+1$  is  $M^{k+1}$



height  $k+1$



may, so at root  $M$  subtrees

heights from 0 to  $k$

and each has at most  $M^k$  leaves

So, at most  $M$  sub trees each with at most  $M^k$  leaves

means, together, at most  $(M)(M^k)$  leaves  
or  $M^{k+1}$  leaves  $\square$

Corollary  $l \leq M^h$

$$\log_M l \leq h$$

$$\text{So } h \geq \lceil \log_M l \rceil$$

So given leaves you have a smallest possible height for trees  $\lceil \log_M l \rceil$

ex

chat text: "Send to 10 people"

$M=10$   
 $l=2000$

you find that 10,000 people get the text and do not send it out.

$$\begin{aligned} n &= i + l \\ n &= M^i + 1 \end{aligned}$$

$\rightarrow$   $(M^i)$   $\leftarrow$  assume this  $M$ -ary tree

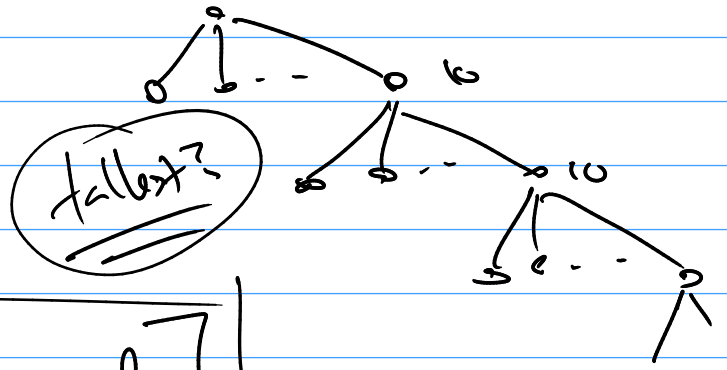
$$\begin{cases} n = i + 10000 \\ n = 10^i + 1 \end{cases}$$

$$\begin{aligned} i + 10000 &= 10^i + 1 \\ 9999 &= 10^i - i \end{aligned}$$

$n = 11,111$   $\leftarrow$  all people  $i = 1,111$   
 $i = 1,111$   $\leftarrow$  servers  
 $l = 10,000$   $\leftarrow$  get do not send

$|E| = n - 1 = 11, 110$  Start and received texts

Steps:  $\rightarrow$  height



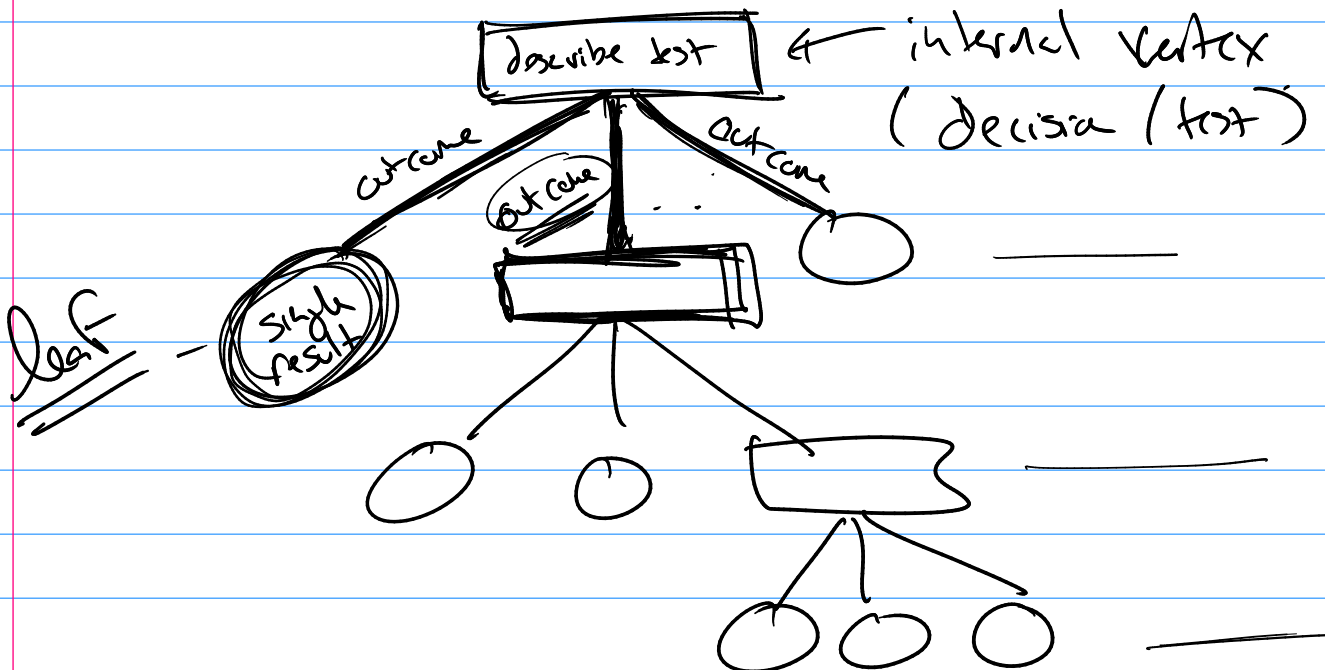
Sketch:

$h \geq \lceil \log_2 n \rceil$

$h = \lceil \log_{10} 10,000 \rceil = \lceil \log_{10} 10^4 \rceil = \lceil 4 \rceil = 4$

Applications:

① Decision Trees

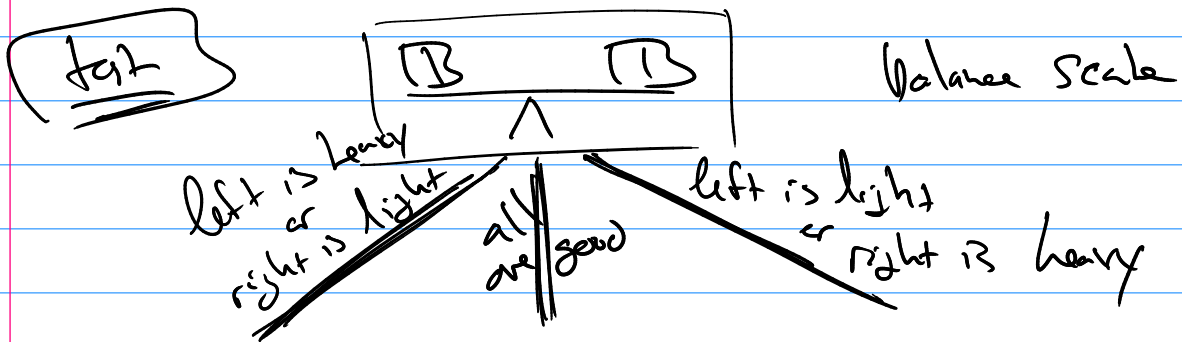


# Decision tree

- ① internal vertex  $\equiv$  test
- ② leaf  $\equiv$  outcome
- ③  $|leaves| = l \Rightarrow$  all possible answers you can get
- ④  $h \geq \lceil \log_m l \rceil$

$m$  is given by the test  
 $l$  is all the possible answers

5 cases. 1 is fake (light? or heavy?)



$$m = 3$$

leaves = outcomes

1L  $\equiv$  coin is light

1H  $\equiv$  coin is heavy

1L, 1H, 2L, 2H, 3L, 3H, 4L, 4H, 5L, 5H

$$l = 10$$

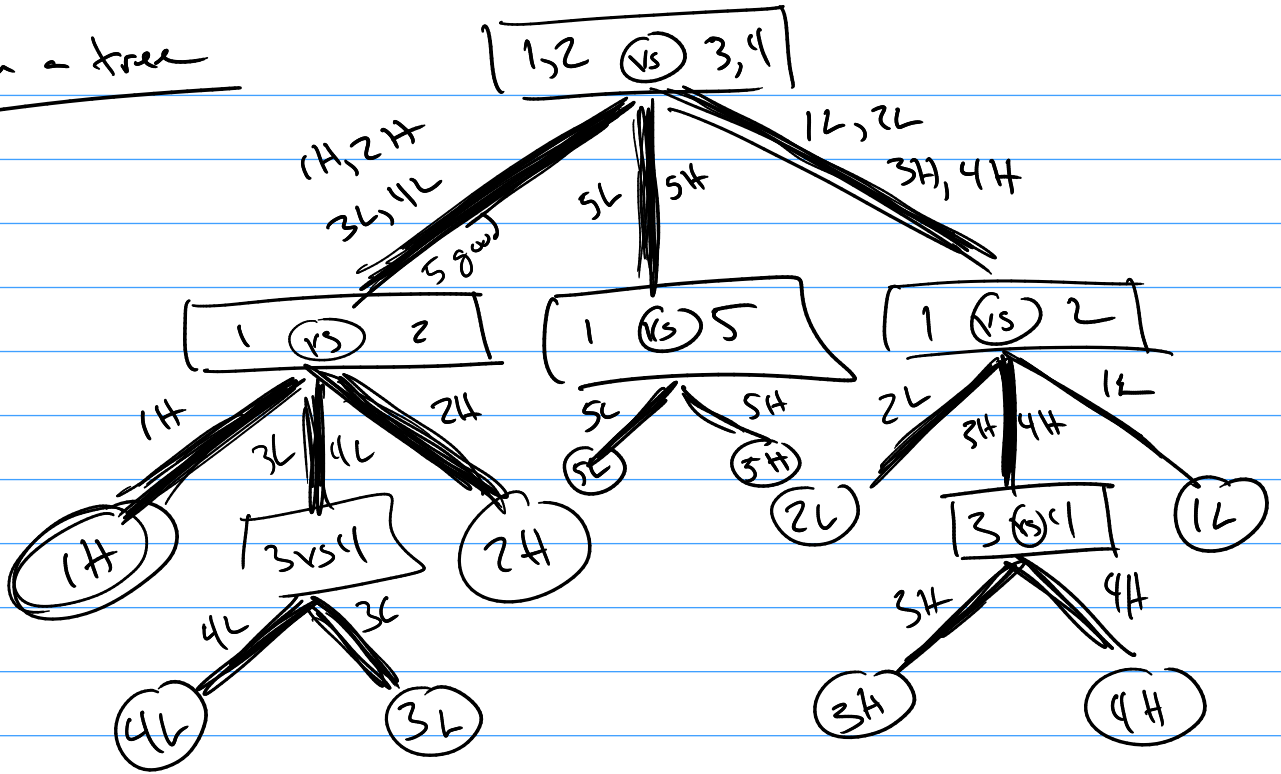
best case?

$$h = \lceil \log_3 10 \rceil$$

$$= 3$$

1L, 1H, 2L, 2H, 3L, 3H, 4L, 4H, 5L, 5H

Make a tree

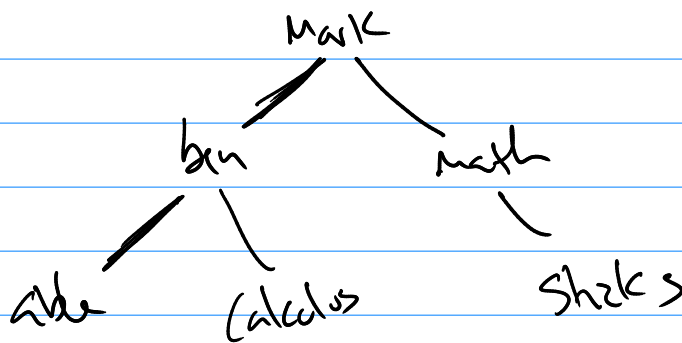


Binary Trees

2-ary

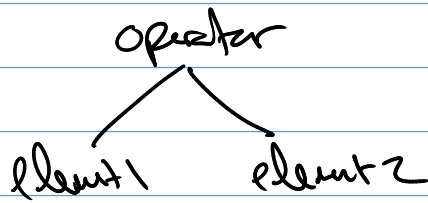
Search tree : - place element at a vertex.  
 - Compare New elements to  
 < vertex  
 less = left  
 more = right

Mark, ~~ben~~, ~~aba~~, ~~math~~, ~~calculus~~, sticks



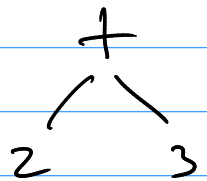
# Prefix, infix, postfix notation

example



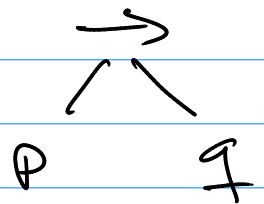
example:

add 2 and 3



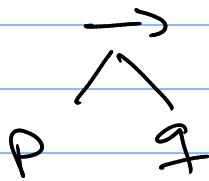
example:

P implies Q



tree traversal → turn vertex labels from tree into a linear form.

pre-order

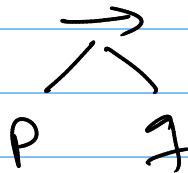


parent, child 1 to M

→, P, Q

prefix

in-order

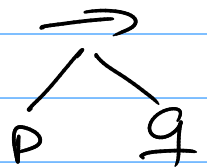


child1, parent, other children

P, →, Q

infix

post-order



child1 to M, parent

P, Q, →

postfix