Math 322

10.1 #3 Inductive Proof

old into new

Forward take old ⇝ make new

Backward find an old part given new
deconstruct

G is tree $\equiv$ $|E| = |V| - 1$

PF **Basis:** $|V| = 1$, tree — • no edges $|E| = 0$

and $|E| \overset{?}{=} |V| - 1$ **Yes**
$|E| = 0$, $|V| - 1 = 0$

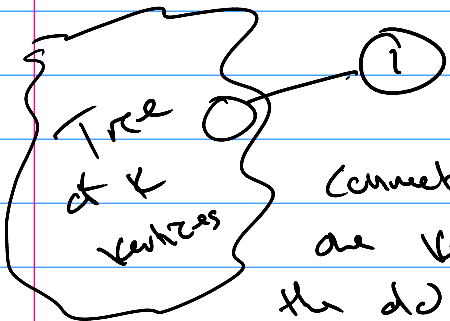
Inductive: **assume** $|V| = 1, 2, \ldots, k$ trees have $|E| = |V| - 1$

Goal: for $|V| = k+1$ then show $|E| = k$ for a tree of $k+1$ vertices

Path #1	Path #2
construct	deconstruct
tree of k vertices ⇝ get a tree of k+1 vert.	start with a tree of k+1 vertices ⇝ find a tree of k vertices in it

given a tree of k vertices that has $k-1$ edges

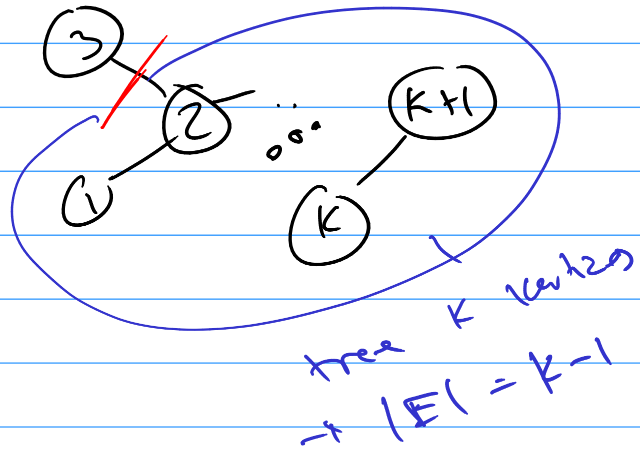
Make a new graph by adding one vertex



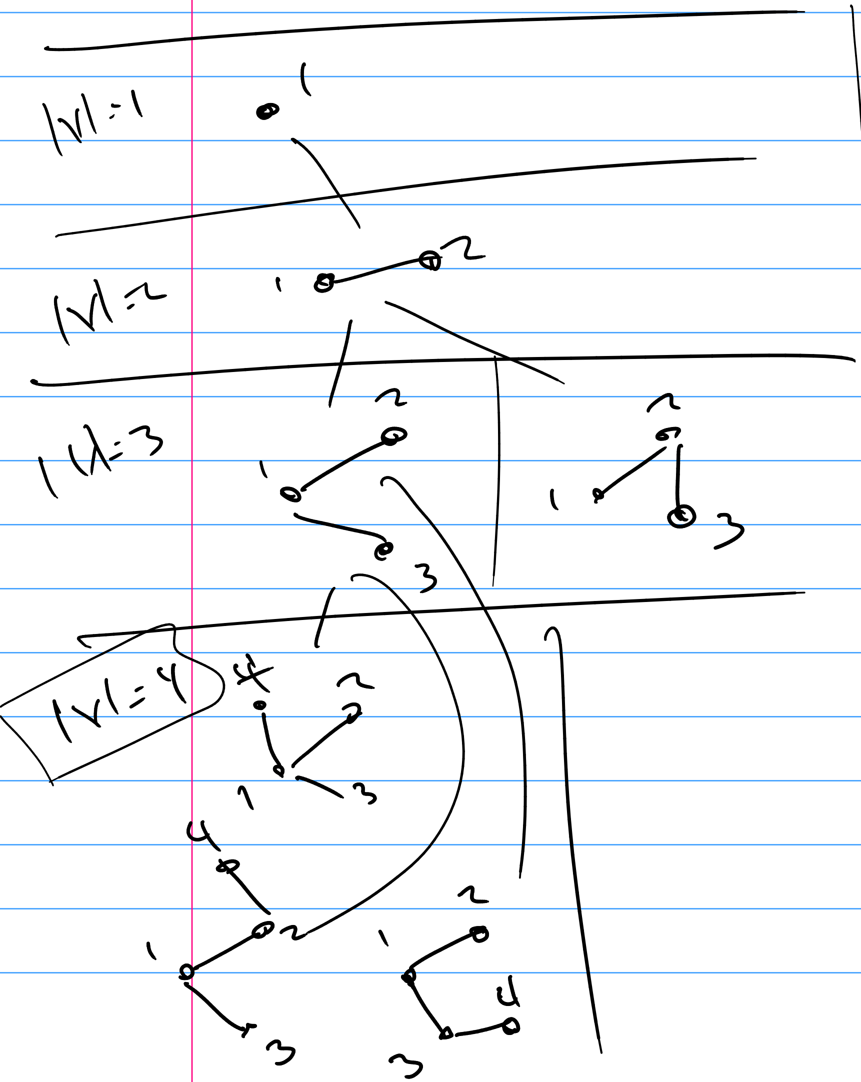
connect to one vertex in the old tree

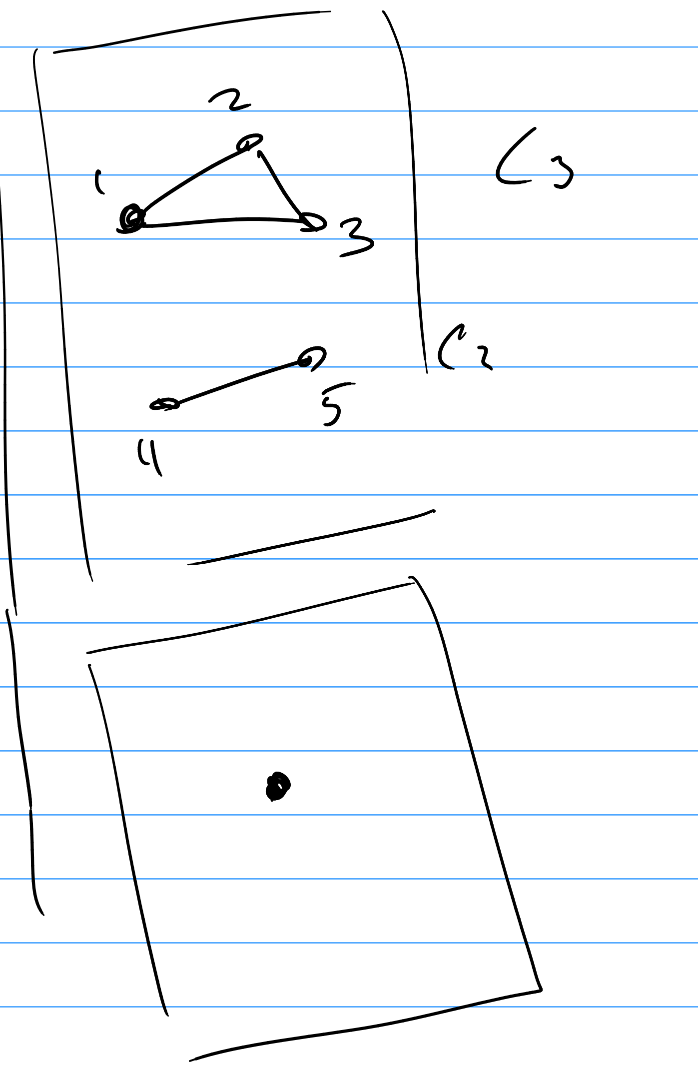
then new graph is a tree with k edges

take a tree of $|V| = k+1$

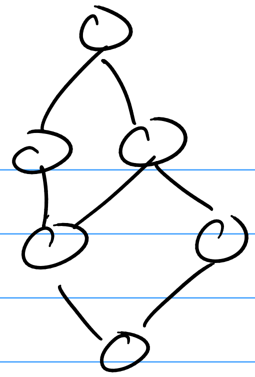


so original had k edges





Boolean Algebra



Posets ; Hasse Diagram

① **Lattice** ≡ Poset where every a, b has a least upper bound $\equiv a \wedge b$ and greatest lower bound $\equiv a \vee b$ ↑

② **Bounded Lattice** ≡ Hasse Diagram has a least and greatest element

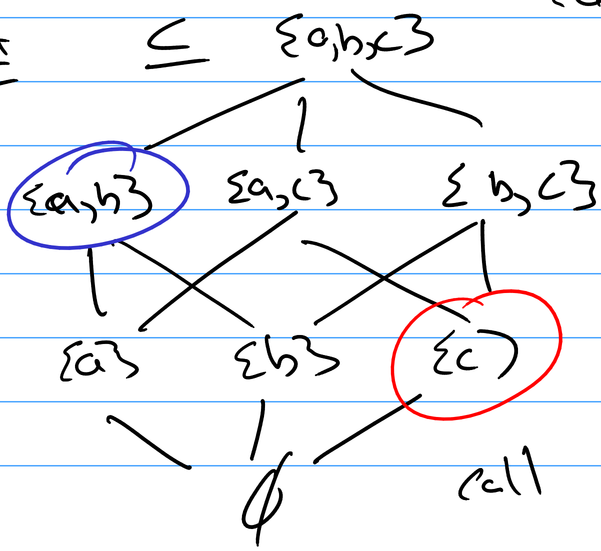
③ **Distributive Lattice** ≡ distributive laws

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$$
$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

④ **Complemented Lattice**

$$P(\{a,b,c\}) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$$

Poset $\subseteq$



greatest lower is $\cap$
least upper is $\cup$

$$\{a,b\} \cap \{c\} = \phi$$
$$\{a,b\} \cup \{c\} = \{a,b,c\}$$

call these elements $\boxed{\text{Complements}}$

If you have a distrib. and complemented Lattice, the complements are unique

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Def: A lattice that is bounded, distrib, and complemented has three operators

1. $a \wedge b$
2. $a \vee b$
3. complement $\bar{a}$

has the following Algebraic Laws...

1. Identity
2. complement
3. associative
4. Commutative
5. Distributive
6. Idempotent
7. Null (domination)
8. Absorption
9. DeMorgan's Laws
10. double complement (or Involution)

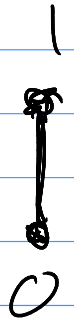
Is a boolean Algebra

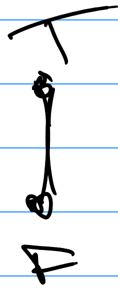
$[B, \vee, \wedge, \bar{\ }]$

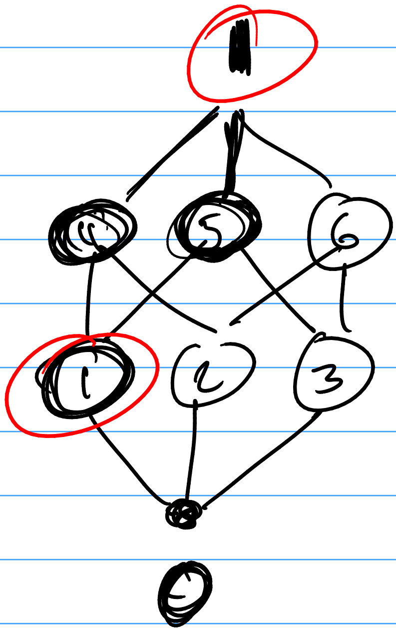
Poset

Lattice with all the above properties

What about the "boolean Algebras" we know







$\mathcal{P}(\{a,b,c\}), \subseteq$

$a' \wedge (b' \vee c')$

$\{a\} \cap (\{a,c\} \cup \{a,b\})$

$\{a\} \cap (\{a,b,c\})$