Math 322 ... Exam 2

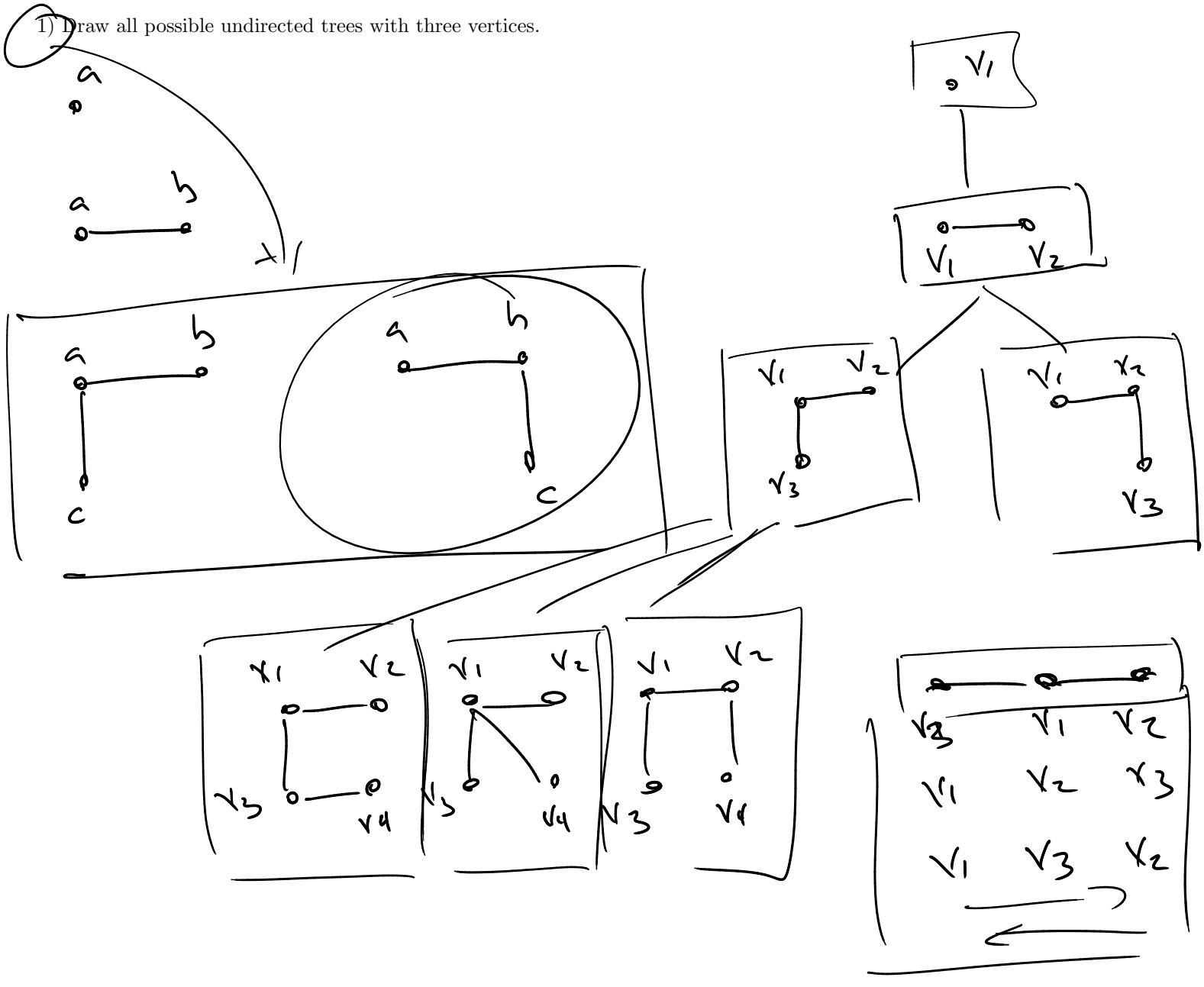
0) Exam Start Time:

0) Name: Key

0) MyWSUid:

0) Please explain all your answers in the exam.

1) Draw all possible undirected trees with three vertices.



2) Prove that if G is a simple undirected graph with no self-loops, then G is a tree if and only if G is connected and $|E| = |V| - 1$.

Basis: of one vertex : a_1

$|V| = 1 \quad |E| = 0$

$0 = 1 - 1$ is true.

Inductive: Inductive hypothesis: assume K vertices and $|E| = K - 1$ and connected and tree

for $K+1$ vertices take tree of K and add one vertex and connect it via one edge.

New graph is a tree and connected

edges of tree of K + added 1

$|E| = (K-1) + 1$

which is $|E| = (K+1) - 1$

tree → connected $|E| = (K+1) - 1$

connected $|E| = (K+1) - 1$ → tree

remove an edge will disconnect

and create two graphs with $|V| = |V_1| + |V_2|$

$G_1 \quad |E_1| = |V_1| - 1$

$G_2 \quad |E_2| = |V_2| - 1$

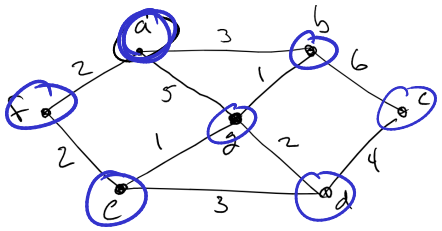
} strong inductive hyp

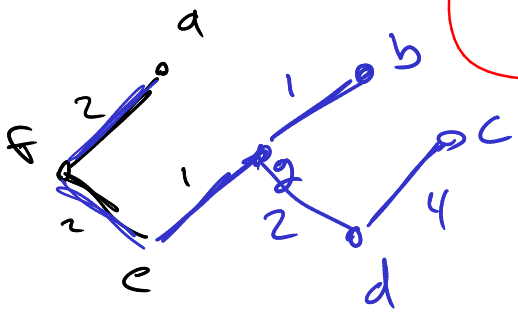
total edges $|E_1| + |E_2| + 1 = |V_1| + |V_2| - 2 + 1$

$= |V| - 1$

$(K+1) - 1$

3) Use Prim's Algorithm to find a spanning tree starting at vertex a.





Set of found

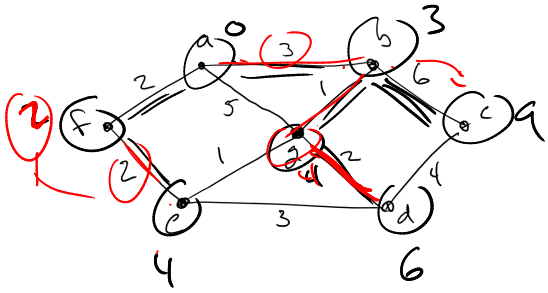
Set of unfound

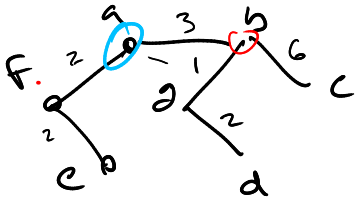
1 a,f
2 f,e
3 e,g
4 g,b
5 g,d
6 d,c

order of edges

total = 12

4) Use Dijkstra's Algorithm to find a spanning tree starting at vertex a





a : 0

a,f : 2

a,b : 3

a,f,e : 4

a,b,g : 4

a,b,g,d : 6

a,b,c : 9

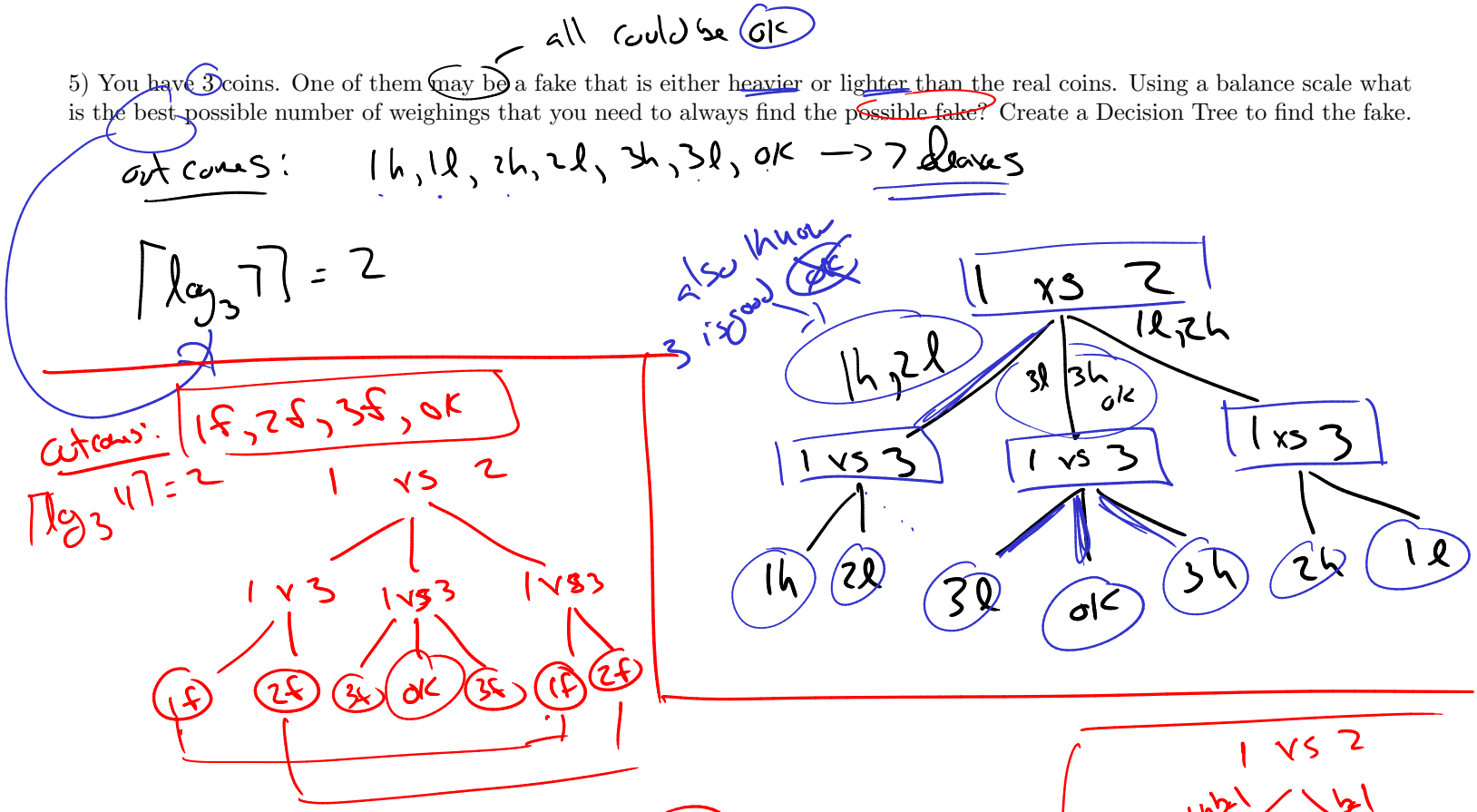
total : 16

set of founds

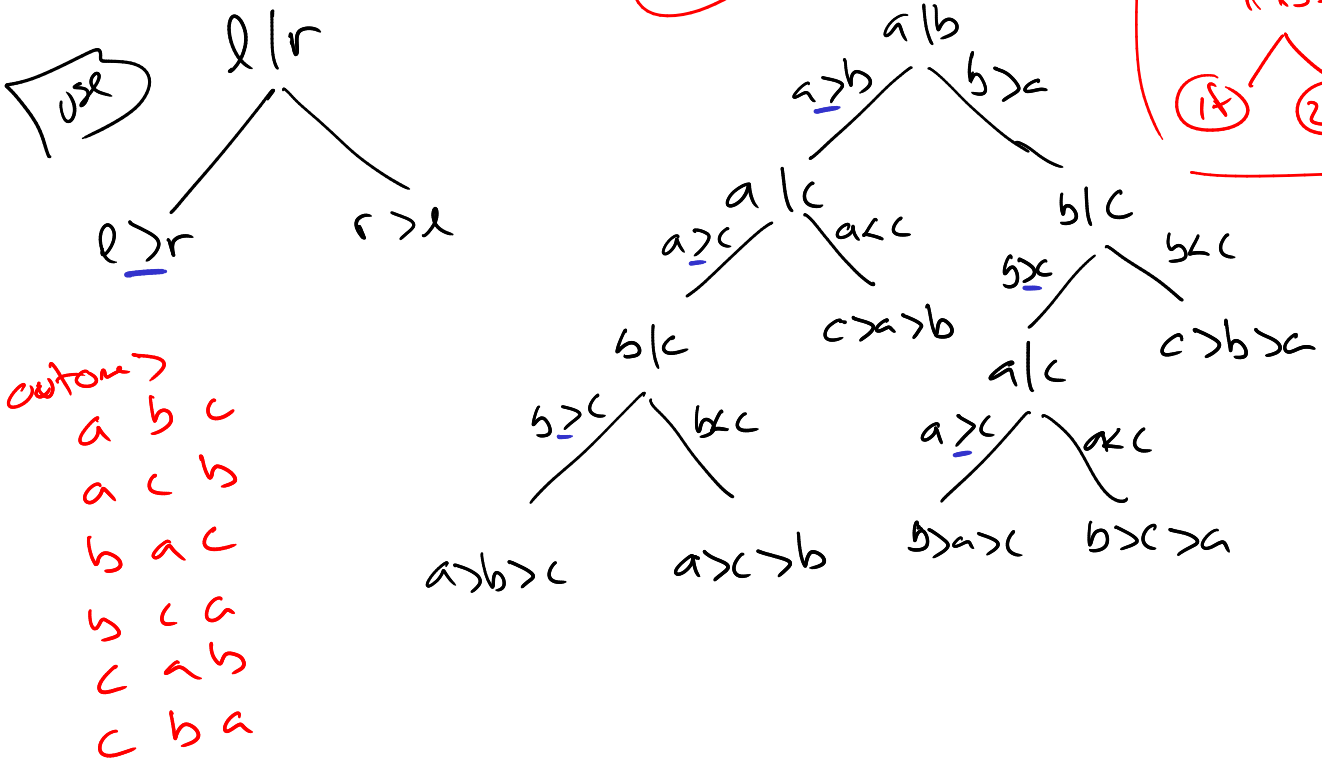
= found has its own cost

set of unfounds

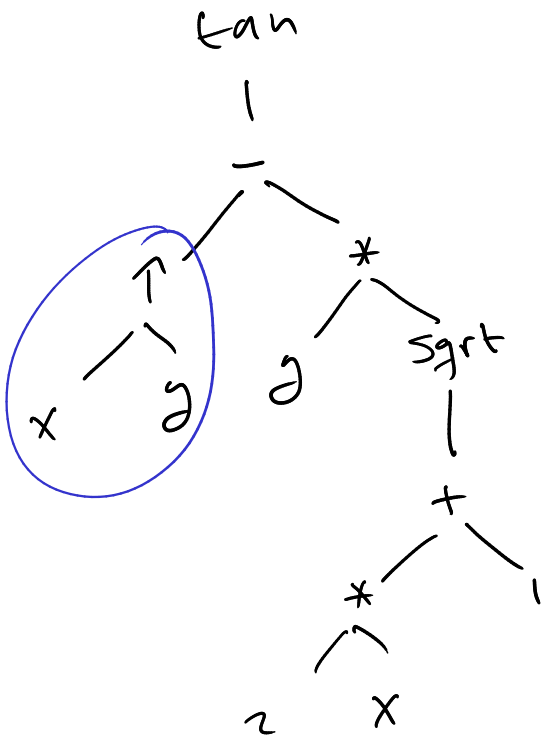
5) You have 3 coins. One of them may be a fake that is either heavier or lighter than the real coins. Using a balance scale what is the best possible number of weighings that you need to always find the possible fake? Create a Decision Tree to find the fake.



6) Construct a Decision Tree to sort three words. a, b, c



7) For the expression $\tan(x^y - y\sqrt{2x+1})$ draw it's given tree and find ...



prefix tan, −, ↑, x, y, *, y, √, +, *, 2, x, 1

postfix x, y, ↑, y, 2, x, *, 1, +, √, *, −, tan

infix (x, ↑, y), −, y, *, √, 2, *, x, +, 1, √, tan

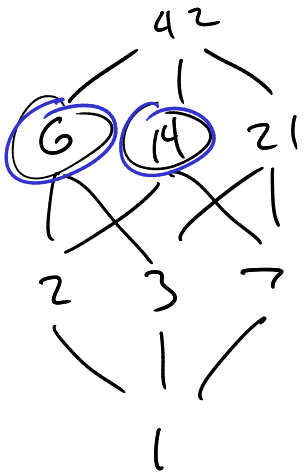
a) The prefix notation

b) The infix notation

c) The postfix notation

8) For the poset $(D_{42}, |)$ where $D_{42} = \{1, 2, 3, 7, 6, 14, 21, 42\}$,

a) Draw the Hasse Diagram for D_{42} with respect to $|$.



lower: 2, 1

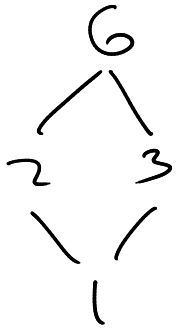
greatest lower: 2

upper: 42

least upper: 42

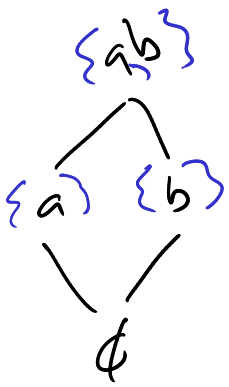
b) Find all lower bounds, upper bounds, the greatest lower bound, and the least upper bound for 6 and 14.

9) For the lattice $(D_6, |)$ where $D_6 = \{1, 2, 3, 6\}$, write its operation table for meet.



$\wedge$	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

10) For the boolean algebra $P(\{a, b\})$ with $\subseteq$, write the operation tables for $\cap, \cup, -$.

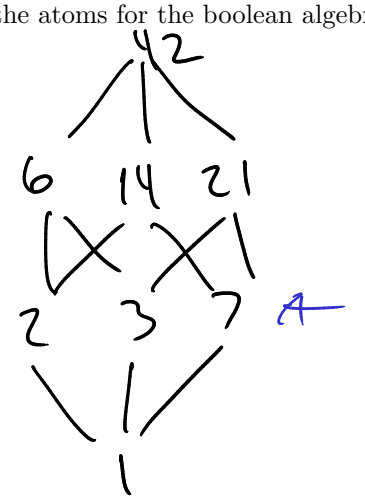


$\cap$	$\emptyset$	a	b	ab
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$
a	$\emptyset$	a	$\emptyset$	a
b	$\emptyset$	$\emptyset$	b	b
ab	$\emptyset$	a	b	ab

$\cup$	$\emptyset$	a	b	ab
$\emptyset$	$\emptyset$	a	b	ab
a	a	a	ab	ab
b	b	ab	b	ab
ab	ab	ab	ab	ab

	$-$
$\emptyset$	ab
a	b
b	a
ab	$\emptyset$

11) Find the atoms for the boolean algebra $(D_{42}, |)$ where $D_{42} = \{1, 2, 3, 7, 6, 14, 21, 42\}$,

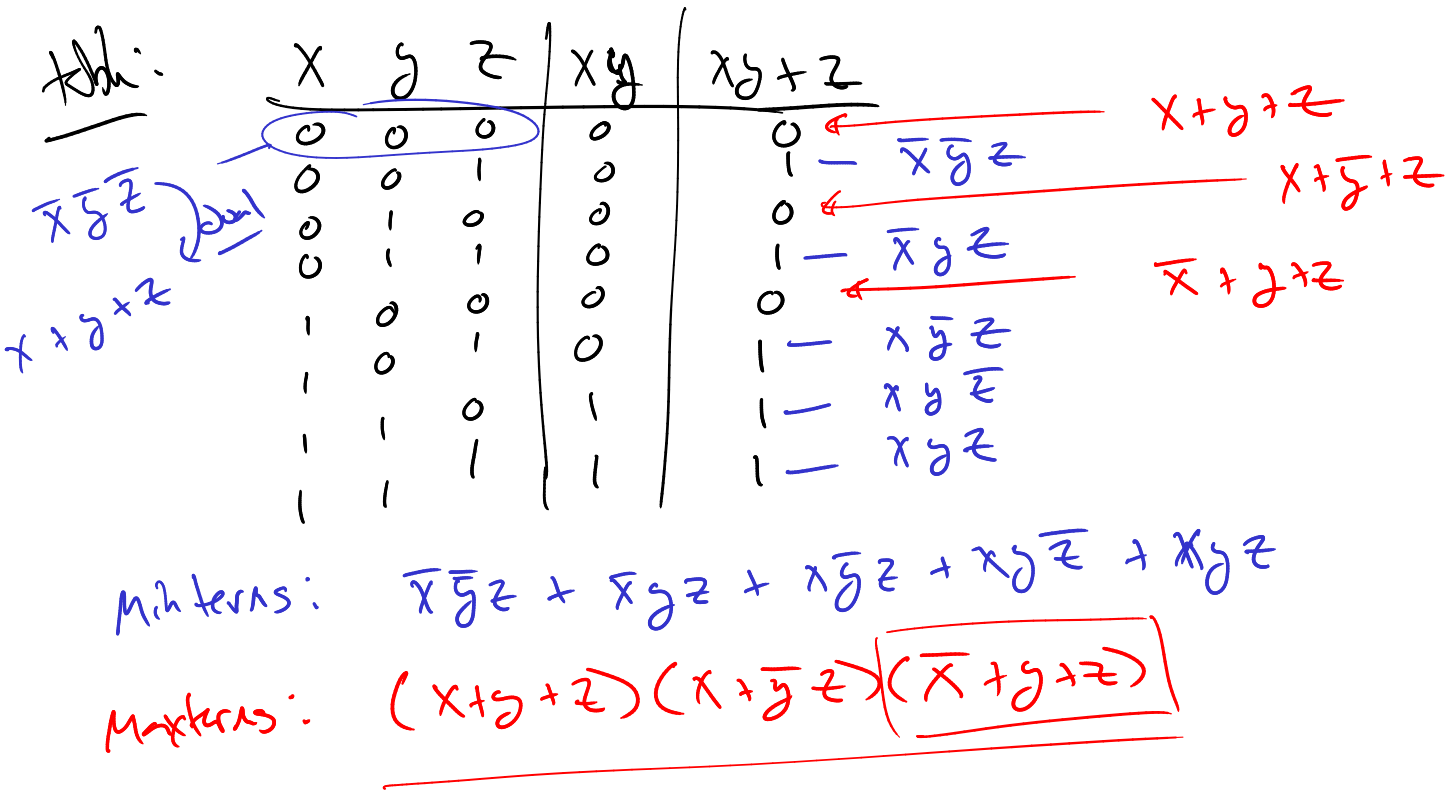


$\wedge$	1	2	3	7	6	14	21	42
2	1	2	1	1	2	2	1	2
3	1	1	3	1	3	1	3	3
7	1	1	1	7	1	7	7	7

So 2, 3, 7 are atoms

Can Make rest of op table

12) Find the minterm and maxterm expansions for $g(x, y, z) = xy + z$



0) What is the time you ended working on the exam and started scanning it?