Math 344

Q's

The errors in measurement are at most $\boxed{8\%}$, so $\overset{dw}{\overset{\parallel}{\left|\frac{\Delta w}{w}\right|}} \le 0.08$ and $\overset{dh}{\overset{\parallel}{\left|\frac{\Delta h}{h}\right|}} \le 0.08$. The relative error in the calculated surface area is

$$\frac{\Delta S}{S} \approx \frac{dS}{S} = \frac{0.1092(0.425w^{0.425-1})h^{0.725}\,dw + 0.1092w^{0.425}(0.725h^{0.725-1})\,dh}{0.1092w^{0.425}h^{0.725}}$$

$$= 0.425\frac{dw}{w} + 0.725\frac{dh}{h}$$

To estimate the maximum relative error, we use $\frac{dw}{w} = \left|\frac{\Delta w}{w}\right| = 0.08$ and $\frac{dh}{h} = \left|\frac{\Delta h}{h}\right| = 0.08 \quad \Rightarrow$

$\frac{dS}{S} = 0.425\,(0.08) + 0.725\,(0.08) = 0.092$. Thus the maximum percentage error is approximately 9.2%

$S = S(w, h)$

$$S = a_1 w^{a_2} h^{a_3}$$

a_1, a_2, a_3 are const.

$$\Delta S \approx dS = \boxed{S_w}\,dw + \boxed{S_h}\,dh$$

find % in S $\quad \frac{\Delta S}{S} \approx \frac{dS}{S}$

$$\frac{dS}{S} = \frac{\cancel{a_1} a_2 h^{a_3} w^{a_2-1}\,dw + \cancel{a_1} a_3 w^{a_2} h^{a_3-1}\,dh}{\cancel{a_1} w^{a_2} h^{a_3}}$$

$$= \frac{a_2 \cancel{h^{a_3}} w^{a_2-1}\,dw}{w^{a_2} \cancel{h^{a_3}}} + \frac{a_3 \cancel{w^{a_2}} h^{a_3-1}\,dh}{\cancel{w^{a_2}} h^{a_3}}$$

$$= (\qquad)\frac{dw}{w}$$

(ex) **Partials** $S = a\boxed{w}^{b} h^{c}$ a, b, c are const.

$$S = S(w, h)$$

$$S_w = \frac{\partial S}{\partial w} = ab\, w^{b-1} h^{c}$$

(ex) $S = \boxed{b^{w} h^{c}}$

$$S_w = \frac{\partial S}{\partial w} = \ln b \; b^{w} h^{c}$$

$$S_h = c\, b^{w} h^{c-1}$$

$$\frac{d}{dx}\{e^{x}\} = e^{x}$$

$$\frac{d}{dx}\{3^{x}\} = \ln 3 \; 3^{x}$$

$$3^{x} = e^{\ln(3^{x})}$$

$$\boxed{e^{x \ln 3}}$$

14.5 **chain rule**

$z = f(x, y)$ but $x = x(t)$

$y = y(t)$

composition $z = f(x(t), y(t))$

$z = f(x,y) \quad x(t), y(t)$

$$\frac{dz}{dt} = \underbrace{z_x \frac{dx}{dt}} + \underbrace{z_y \frac{dy}{dt}}$$

Many many variables?

$$u = f(x_1, x_2, x_3, \dots, x_n)$$

and $\boxed{x_1(t_1, t_2, \dots, t_m)}$

$x_2(t_1, t_2, \dots, t_m)$

$\vdots$

$$u_{t_1} = u_{x_1} \frac{\partial x_1}{\partial t_1} + u_{x_2} \frac{\partial x_2}{\partial t_1} + \dots + u_{x_n} \frac{\partial x_n}{\partial t_1}$$

$$u_{t_2} = u_{x_1} \frac{\partial x_1}{\partial t_2} + u_{x_2} \frac{\partial x_2}{\partial t_2} + \dots + u_{x_n} \frac{\partial x_n}{\partial t_2}$$

$\vdots$

ex $\quad S = 3w^4 h^2 q^{1/2} \qquad S = S(w, h, q)$

but $\quad w = l + t^2 \qquad w(l,t)$

$h = lt \qquad h(l,t)$

$q = l^2 - t \qquad q(l,t)$

$$S_\ell = \boxed{S_w}\boxed{w_\ell} + \boxed{S_h}\boxed{h_\ell} + \boxed{S_g}\boxed{g_\ell}$$

$$S = 3w^4h^2g^{1/2}$$

but

$$w = \ell + t^2$$
$$h = \ell t$$
$$g = \ell^2 - t$$

$$S_\ell = 12w^3h^2g^{1/2} \cdot (1)$$
$$+ 6w^4hg^{1/2} \cdot (t)$$
$$+ \frac{3}{2}w^4h^2g^{-1/2} \cdot (2\ell)$$

try to find S_t?

Application Use Chain Rule for implicit deriv.

(ex) $3x^2 + xy = y^2$ $\quad \frac{dy}{dx} = ?$ $\quad y = f(x)$

$$6x + \left(y + x\frac{dy}{dx}\right) = 2y\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{6x + y}{2y - x}$$

(ex) $x^2 + y^2 + z^2 = (xyz)$ $\quad z = f(x,y)$

$$z_x \rightarrow \quad 2x + 2z\frac{\partial z}{\partial x} = yz + xy\frac{\partial z}{\partial x}$$

$$\frac{\partial z}{\partial x} = \frac{2x - yz}{xy - 2z}$$

by new chain rule...

any implicit equation can be made

$y = f(x)$ to $F(x,y) = 0$

ex $3x^2 + xy = y^2$

$$\underbrace{\boxed{3x^2 + xy - y^2}}_{F} = 0$$

$$F_x \frac{dx}{dx} + F_y \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{F_x}{F_y} = -\frac{6x+y}{x-2y} = \boxed{\frac{6x+y}{2y-x}}$$

ex

or $z = f(x,y)$ implicitly?

$$x^2 + y^2 + z^2 = xyz$$

$$\underbrace{\boxed{x^2 + y^2 + z^2 - xyz}}_{F} = 0$$

$$F = 0$$

$\frac{\partial}{\partial x}$ $$F_x \frac{\partial x}{\partial x} + F_y \frac{\partial y}{\partial x} + F_z \frac{\partial z}{\partial x} = 0$$

$$\frac{\partial x}{\partial x} = 1, \quad \frac{\partial y}{\partial x} = 0$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$$

Ex $$\underbrace{x^2+y^2+z^2-xyz}_{F} = 0$$

$$z_x = -\frac{2x-yz}{2z-xy} = \boxed{\frac{2x-yz}{xy-2z}}$$

Similarly

$$z_y = \boxed{\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}}$$