Math 344

$f(x,y) \xrightarrow{\text{partials}}$ $f_x \to f_{xx}, f_{xy} \to$

$f_y \to f_{yx}, f_{yy} \to$

1st 2nd etc

$f(x,y,z) \rightsquigarrow$ $f_x \longrightarrow f_{xx}, f_{xy}, f_{xz}$

$f_y \to$ $\to$

$f_z \to$ etc

1st 2nd

Gradient $\nabla f = \langle f_x, f_y \rangle$ if $f(x,y)$

$\nabla f = \langle f_x, f_y, f_z \rangle$ if $f(x,y,z)$

Apps for ∇f?

① $D_u(f) = \nabla f \cdot u$

$u = \langle a,b \rangle$
or
$u = \langle a,b,c \rangle$

② Max Value of $D_u(f)$ is in the direction of ∇f and is $|\nabla f|$

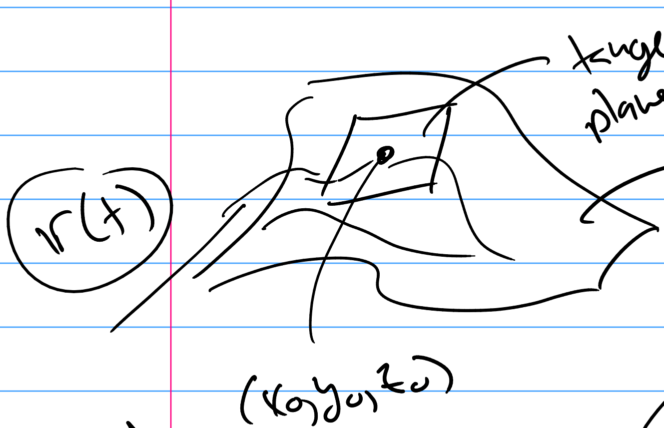
So ∇f is direction of fastest increase

and the value of fastest inc. is

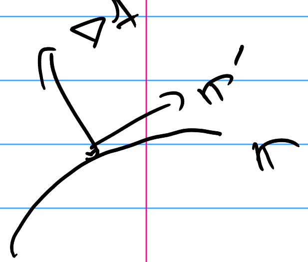
$$|\nabla f|$$

(3) $g = f(x,y,z) \rightsquigarrow$ level surfaces

$$f(x,y,z) = K$$



$f(x,y,z) = K$ $\qquad r(t) = \langle x(t), y(t), z(t) \rangle$



$$\underline{\underline{f(x,y,z)}} = \underline{\underline{K}} \qquad x(t), y(t), z(t)$$

$$f_x \cdot \frac{dx}{dt} + f_y \cdot \frac{dy}{dt} + f_z \cdot \frac{dz}{dt} = 0$$

$$\langle f_x, f_y, f_z \rangle \cdot \langle x', y', z' \rangle = 0$$

$$\nabla f \cdot r' = 0$$

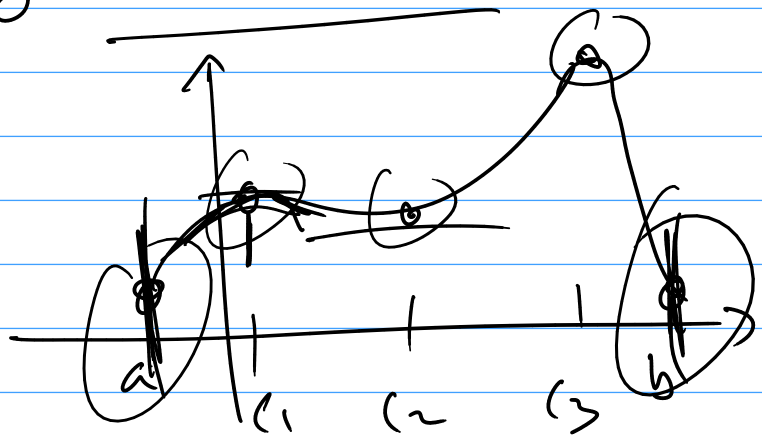
eqn of tangent plane to level surface @ (x_0, y_0, z_0)

$$\boxed{f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0)(z-z_0) = 0}$$

~~App~~

Finding extreme Values

Calc 1



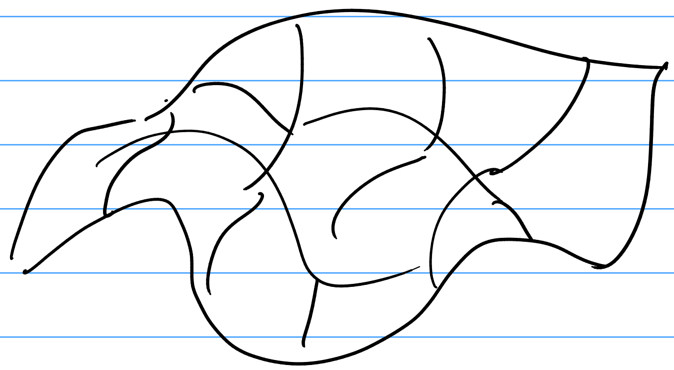
Now in Calc 3

$z = f(x,y)$

$\boxed{14.7}$

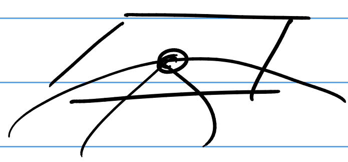
local max/min

abs. max/min



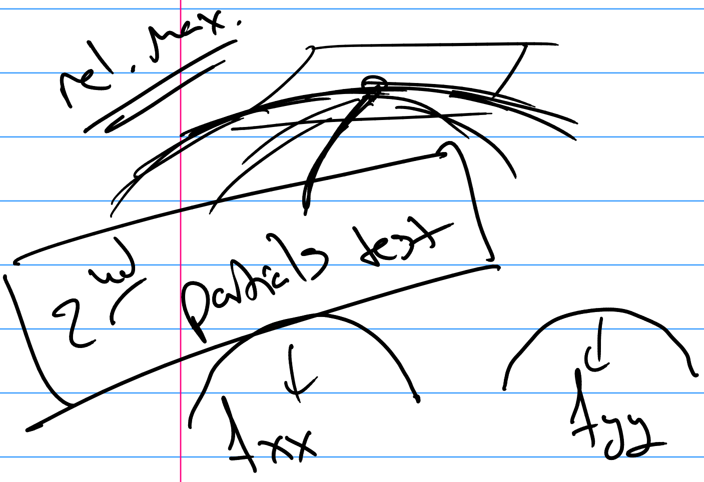
$\boxed{\text{thm}}$ if $z = f(x,y)$ has local max/min @ (a,b) and f_x, f_y exist and cont.

then $f_x(a,b) = 0$
$f_y(a,b) = 0$



call (a,b) a $\boxed{\text{critical point}}$ for $f_x(a,b)=0$, $f_y(a,b)=0$

after you find all (a,b) critical points



let $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$

so $D = f_{xx} f_{yy} - (f_{xy})^2$

$$D(a,b) = f_{xx}(a,b)\, f_{yy}(a,b) - \left(f_{xy}(a,b)\right)^2$$

(1) if $D(a,b) > 0$ $\rightarrow$ (a) $f_{xx} > 0 \Rightarrow$ local min @ (a,b)

$\rightarrow$ (b) $f_{xx} < 0 \Rightarrow$ local max @ (a,b)

(2) if $D(a,b) < 0 \rightarrow$ no local max/min

it is a saddle point.

(3) if $D(a,b) = 0 \Rightarrow$ test fails.

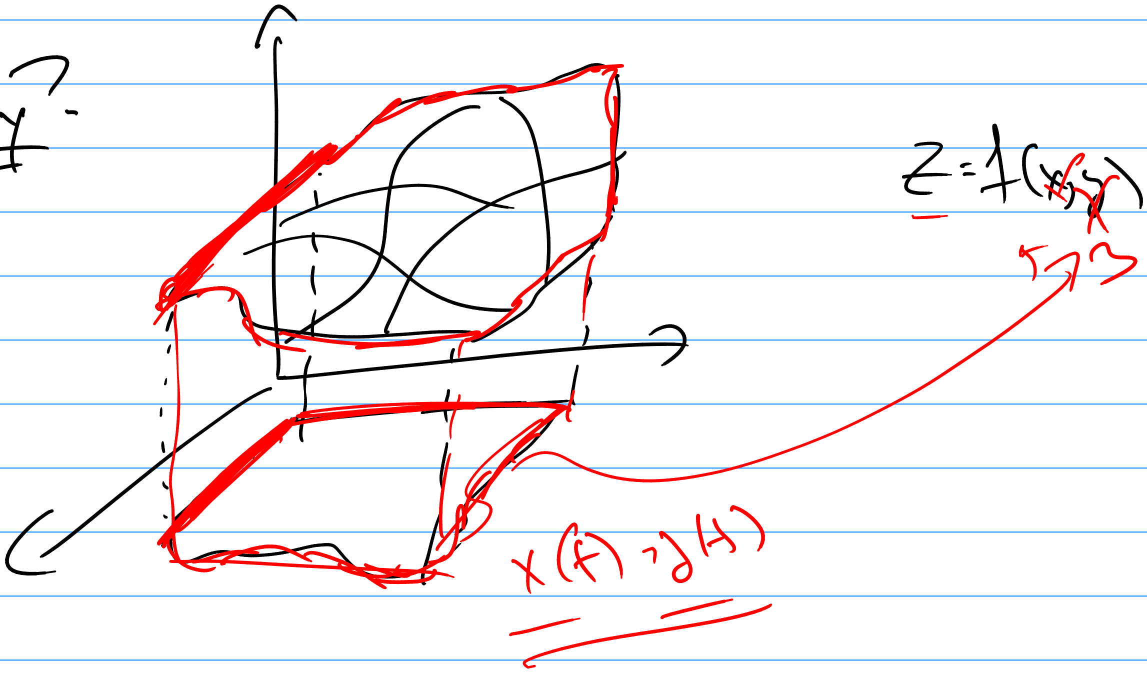
$\boxed{\text{Th}^m}$ $z = f(x,y)$ is cont. on a closed region then it has an absolute max and min.

Where? - (a) on boundry

or

(b) @ rel. extrema

Boundry? -



Exam 1 $\quad$ 0/100 $\quad$ 120 110

new $\left(10, r(t) = \right.$