Math 344

Q's $D_u(f) = \nabla f \cdot u$ ← unit vector

direction of $\langle 1,2,3\rangle$ → $u = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$

$$D_u(f) = \frac{1}{\sqrt{14}}\left(\boxed{\nabla f \cdot \langle 1,2,3\rangle}\right)$$

in direction of v $\quad D_u(f) = \frac{1}{|v|} \nabla f \cdot v$

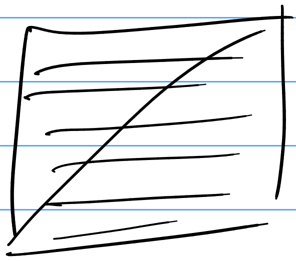
ch 14

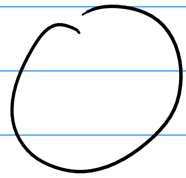
ch 15

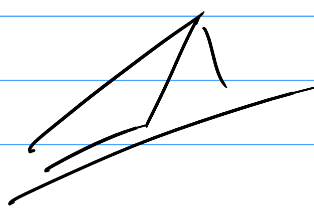
$$\boxed{u = f(x_1, x_2, \ldots, x_n)}$$

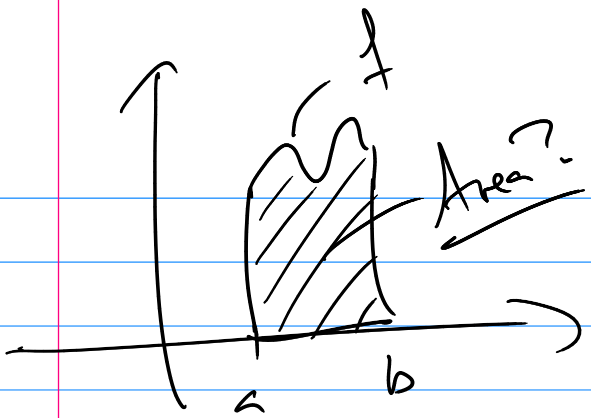
→ Infinite Sums

Calc 1 $\quad$ Area

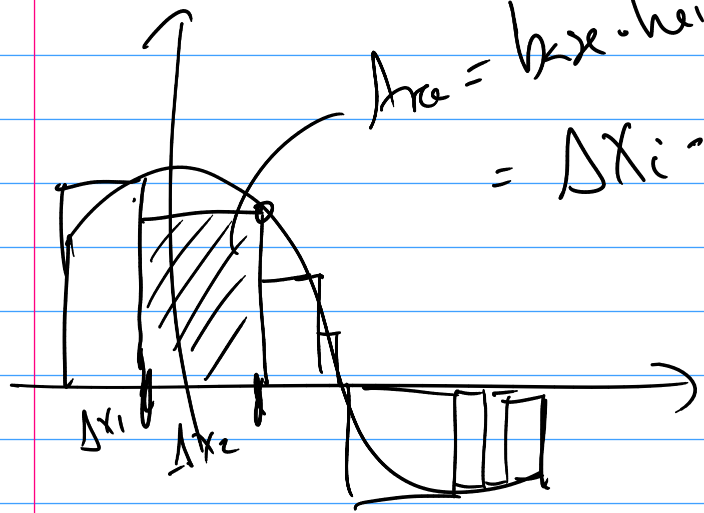








$$\int_a^b f(x)\,dx$$



Area = base · height

$= \Delta x_i \cdot f(x_i^*)$

$A \approx$ Sum of rectangles

Make largest $\Delta x_i \to 0$

$$A = \lim_{\max \Delta x_i \to 0} \sum_{i=1}^{n} f(x_i^*)\,\Delta x_i$$

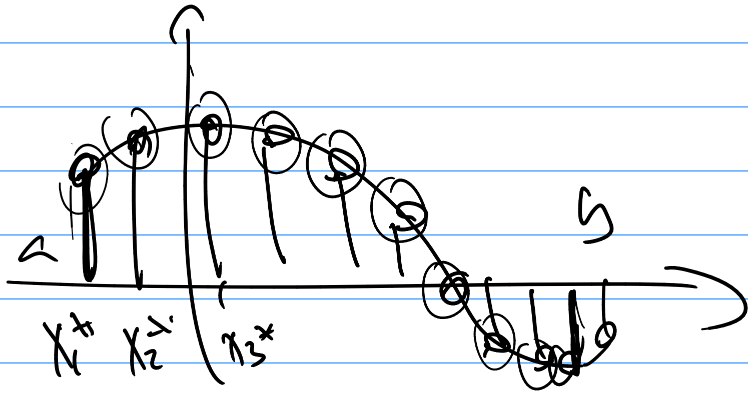
Uniform partition $\Delta x_i = \Delta x = \frac{b-a}{n}$

so $\max \Delta x_i \to 0$ means $\lim_{n\to\infty}$

$$A = \lim_{n\to\infty} \sum_{i=1}^{\infty} f(x_i^*)\,\Delta x \qquad \frac{b-a}{n}$$

$$A = \lim_{n\to\infty} (b-a)\,\frac{\sum_{i=1}^{n} f(x_i^*)}{n}$$

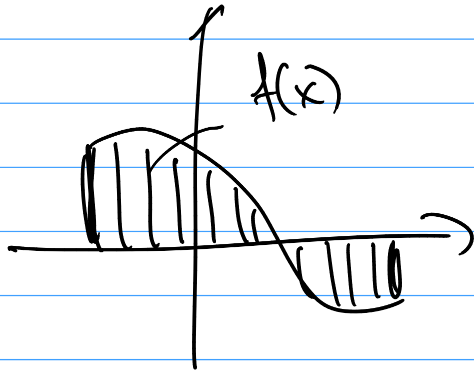
$$A = \lim_{n\to\infty} (b-a) \frac{f(x_1^*) + f(x_2^*) + \dots + f(x_n^*)}{n}$$



$$A \approx (b-a) \frac{\sum_{i=1}^{n} f(x_i^*)}{n}$$

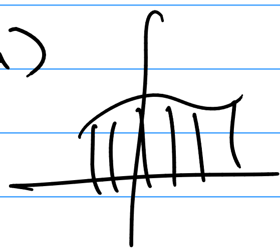
$A \approx$ (width) (average height)

$$\underline{\underline{\text{as}}} \quad \lim_{n\to\infty} \text{f(w)}_{x^*} \int_a^b f(x)\,dx \quad \text{is}$$



$u = f(x_1, x_2, \dots, x_n)$

$u = f(x_1)$



Infinite Sum?

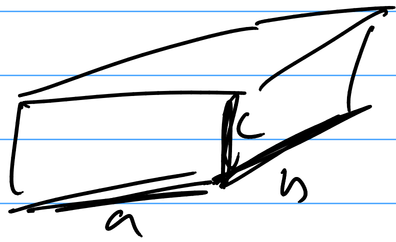
$\boxed{u = f(x,y)}$

15.1 Volume over a rectangular region

$a \le x \le b$

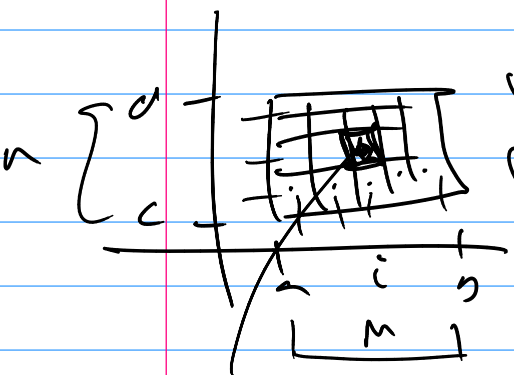
$c \le y \le d$

Vol?



$V = (\text{area of base})(\text{height})$

$V = abc$



unif. partition =

$\Delta x = \frac{b-a}{m}$ $\quad \Delta y = \frac{d-c}{n}$

$\text{base} = \Delta A = \frac{(b-a)(d-c)}{mn}$

Point in here (x^*_{ij}, y^*_{ij})

$V = (\Delta A)\, f(x^*_{ij}, y^*_{ij})$

$V \approx$ Sum of the $(\Delta A)(f(x^*_{ij}, y^*_{ij}))$

$$V \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(x^*_{ij}, y^*_{ij})\, \Delta A$$

$$V = \lim_{n,m \to \infty} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x^*_{ij}, y^*_{ij})\, \Delta A$$

$$V = \iint_R f(x,y)\, dA$$

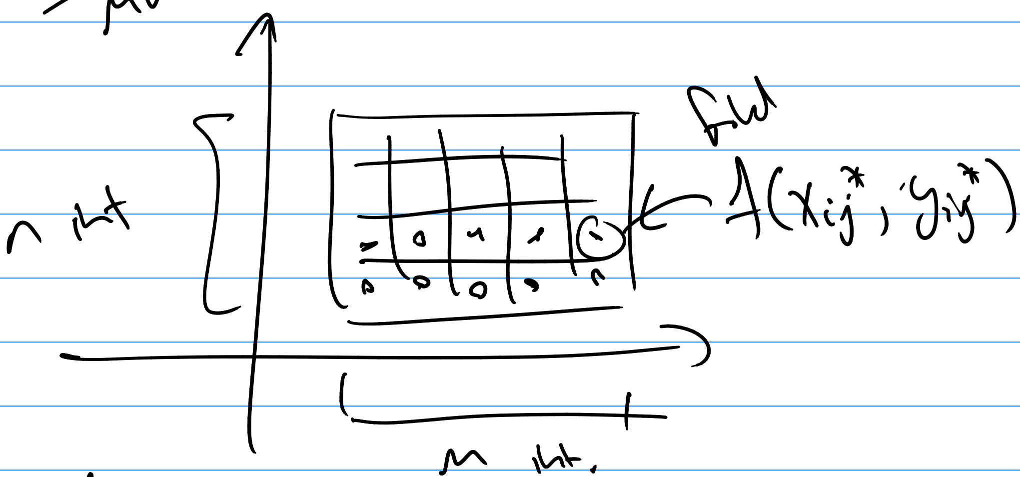
Approx take fixed m, n

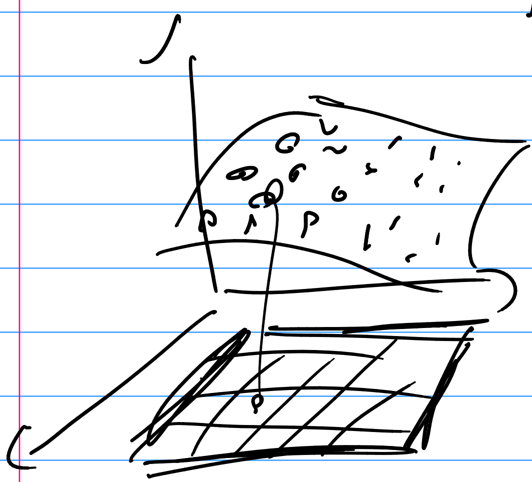
$$V \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\, \Delta A$$

Uniform:

$$V \approx (b-a)(d-c)\, \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)}{mn}$$

$$\Delta A = \frac{(b-a)(d-c)}{mn}$$

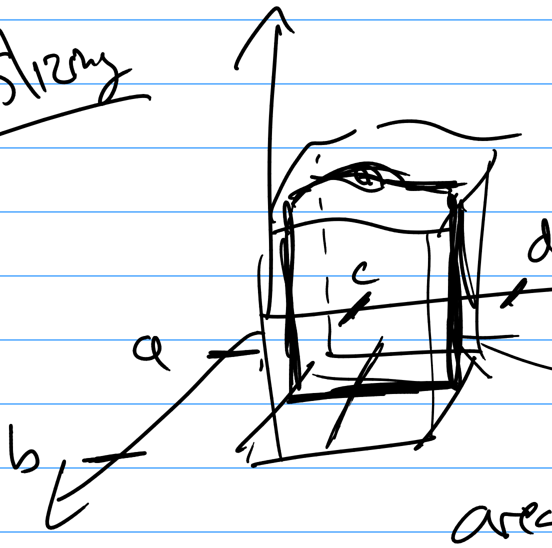




$$V \approx \underbrace{(b-a)(d-c)}_{\text{area base}} \cdot \left(\text{ave height}\right)$$

Set exact ans?

Volume by slicing



$$V = \int_a^b (\text{area of slice})\, dx$$

area of slice

$$= \int_c^d f(x,y)\, dy$$

slice $\perp$ to x-axis

So $$V = \iint_R f(x,y)\, dA = \int_a^b \left[\int_c^d f(x,y)\, dy \right] dx$$

or slice $\perp$ to y-axis

$$V = \int_c^d \left[\int_a^b f(x,y)\, dx \right] dy$$

So $$\iint_R f(x,y)\, dA = \int_a^b \left[\int_c^d f\, dy \right] dx \checkmark$$

Iterated Integration

$$= \int_c^d \left[\int_a^b f\, dx \right] dy \checkmark$$

$$\iint_R f\,dA = \int_a^b \left(\int_c^d f\,dy \right) dx$$

$$= \int_c^d \int_a^b f\,dx\,dy$$