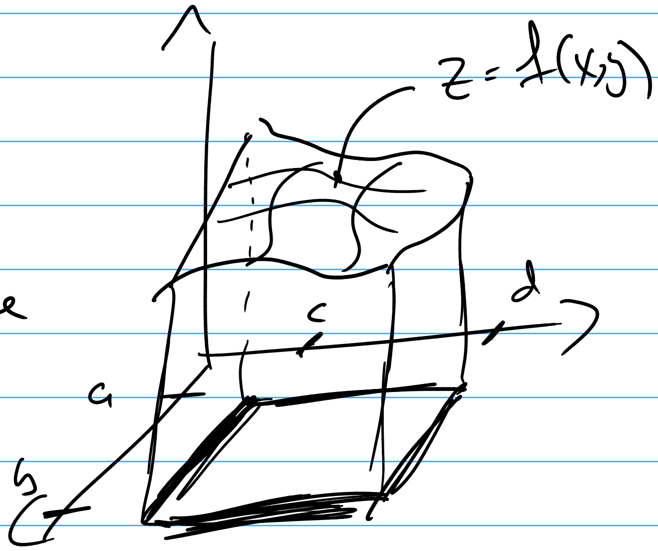
Math 344

Ch 15 Integration on $u = f(x_1, x_2, \ldots, x_n)$

15.1 $z = f(x,y)$

$$\iint_R f(x,y)\,dA = \text{Volume}$$



* R is rectangular

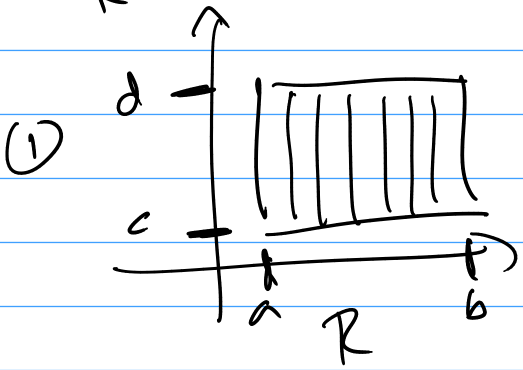
Volume $\approx \underbrace{[(b-a)(d-c)]}_{\text{area of } R}$ (a average height)

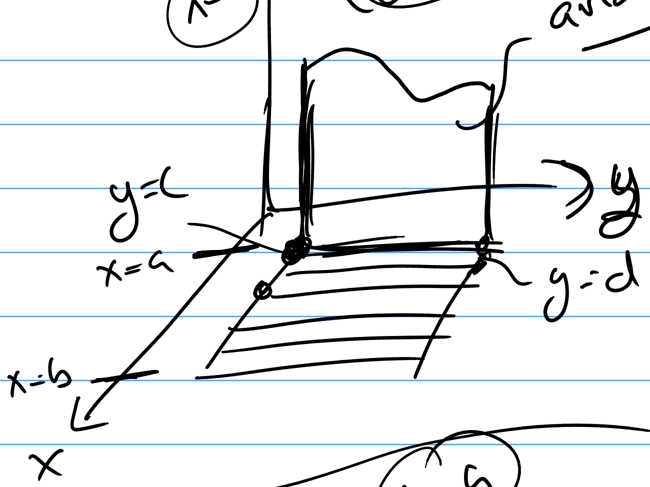
① grid R
② find your points in the grid (x^*_{ij}, y^*_{ij})
③ $f(x^*_{ij}, y^*_{ij})$ = height

Analytically? (Vol. by slicing)

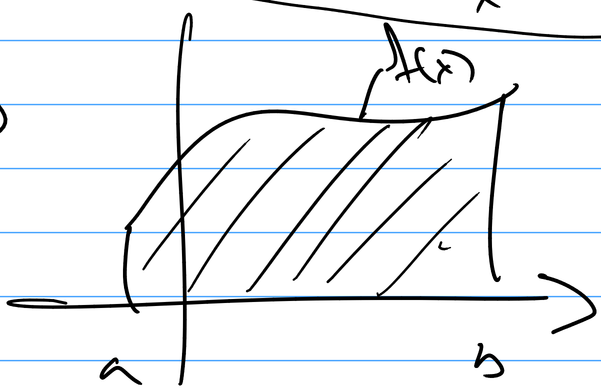
$$Vol = \iint_R f(x,y)\,dA = \int_{x=a}^{x=b}\left(\int_{y=c}^{y=d} f(x,y)\,dy\right)dx$$

area of slice



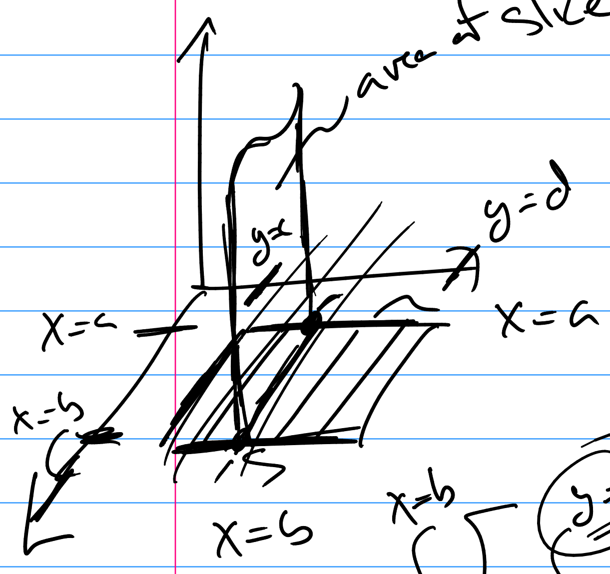


remember calc I



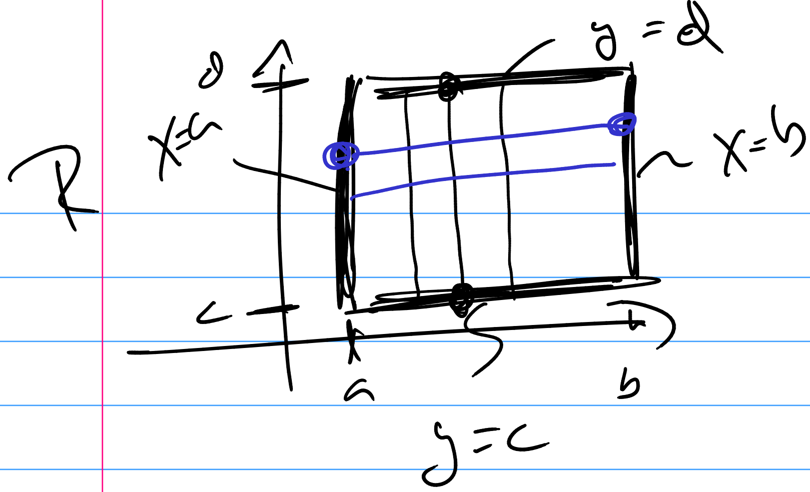
$$A = \int_{x=a}^{x=b} \boxed{f(x)}\,dx$$

$= \boxed{\#}$ 10 x's



$$Vol = \int_{y=c}^{y=d}\left[\int_{x=a}^{x=b} f(x,y)\,dx\right]dy$$

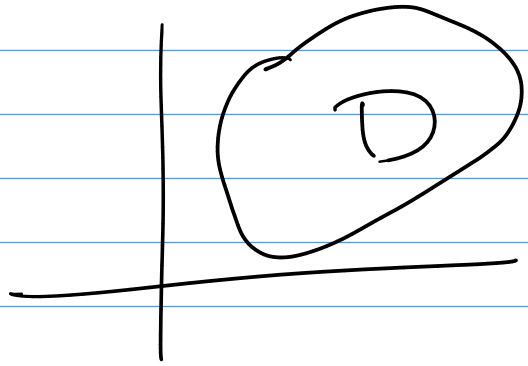
So $$Vol = \int_{x=a}^{x=b}\left[\int_{y=c}^{y=d} f(x,y)\,dy\right]dx = \int_{y=c}^{y=d}\left[\int_{x=a}^{x=b} f(x,y)\,dx\right]dy$$



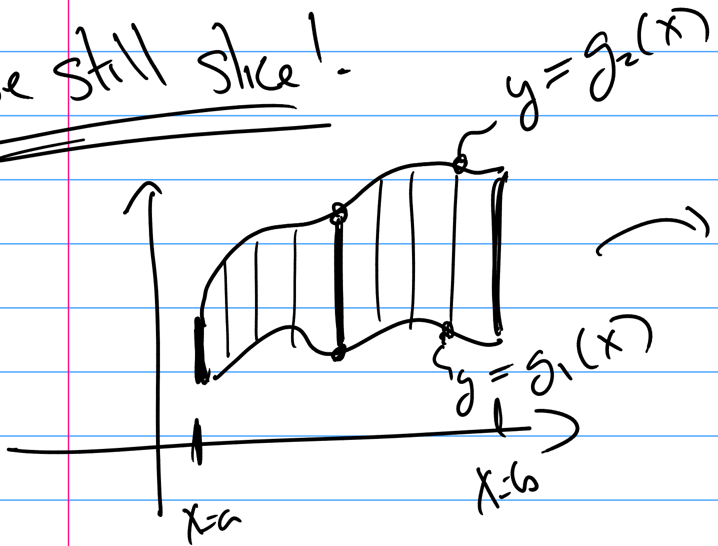
$\boxed{15.2}$

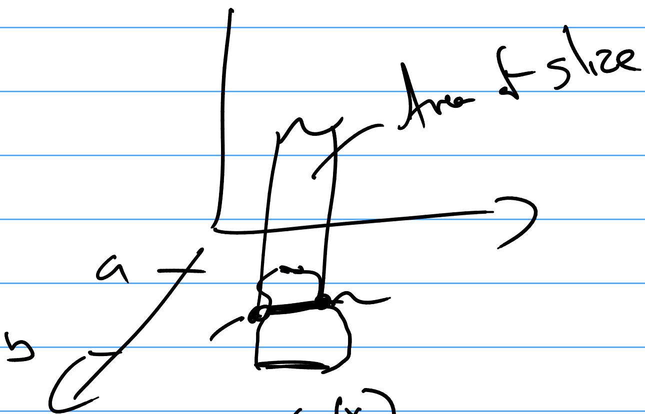
$$\text{Vol} = \iint_{D} f(x,y)\, dA$$

not rectangular?



we still slice!

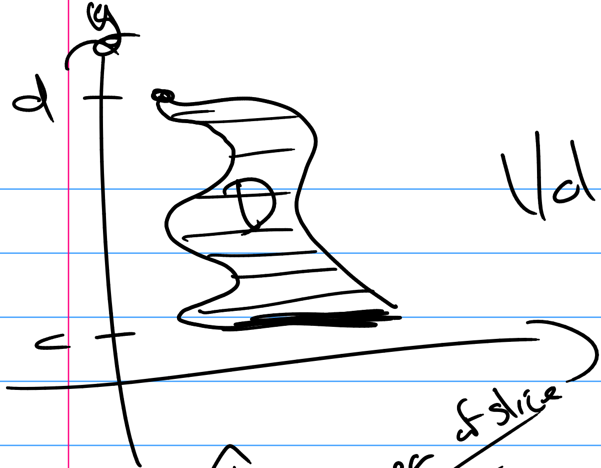




$$V = \int_{x=a}^{x=b} \left[\int_{y=g_1(x)}^{y=g_2(x)} f(x,y)\, dy \right] dx$$

only numbers

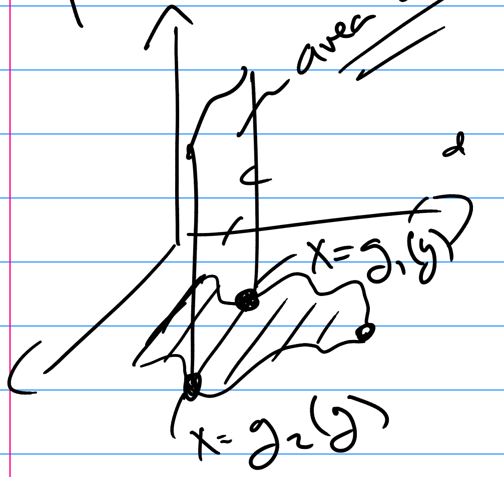
only x's and const.



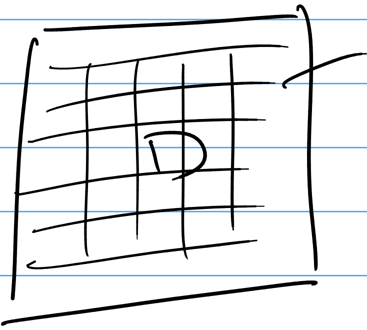
$$\text{Vol} = \int_{y=c}^{y=d} \left[\int_{x=g_1(y)}^{x=g_2(y)} f(x,y)\, dx \right] dy$$

$y=c$: numbers only

x = y's and const

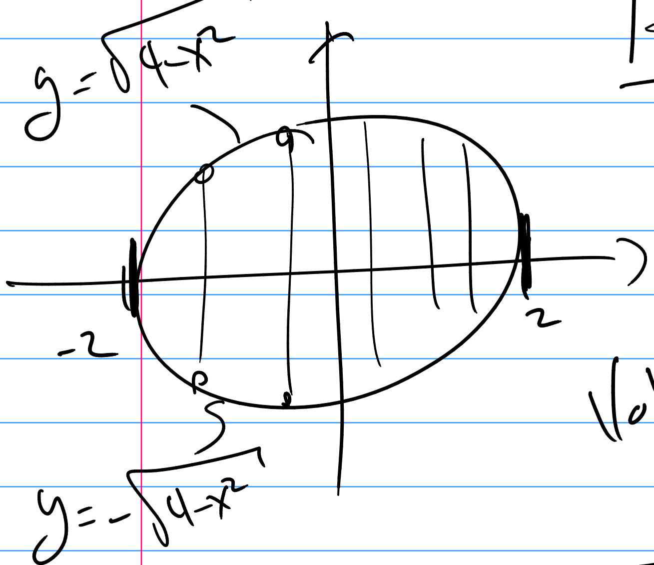


how to slice?



doesn't matter (both are easy)

see chapter 15.1

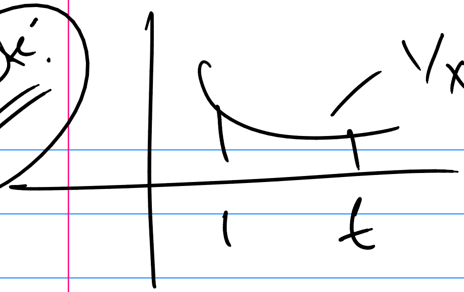


$x^2+y^2=4$

$$\text{Vol} = \int_{x=-2}^{x=2} \left[\int_{y=-\sqrt{4-x^2}}^{y=\sqrt{4-x^2}} f(x,y)\, dy \right] dx$$

$$\text{Vol} = \int_{y=-2}^{y=2} \left[\int_{x=-\sqrt{4-y^2}}^{x=\sqrt{4-y^2}} f(x,y)\, dx \right] dy$$

Note:



$$\int_{x=1}^{x=t} \frac{1}{x^2}\,dx$$

$$-\frac{1}{x}\Big|_{x=1}^{x=t} = \left[-\frac{1}{t} - \frac{-1}{1}\right]$$

$$= 1 - \frac{1}{t}$$

Note:

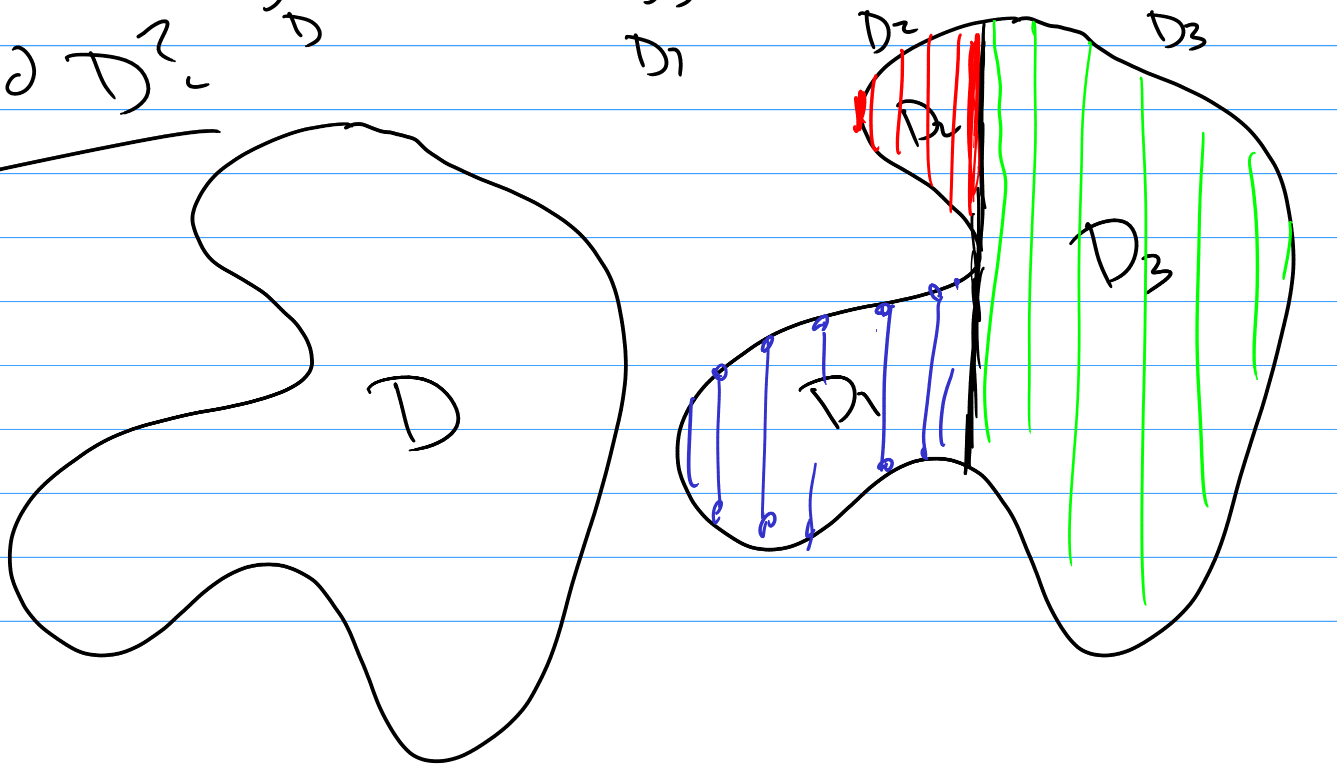
$$\int_{x=-y^3+1}^{x=y^2+3y} \frac{1}{x^2}\,dx = -\frac{1}{x}\Big|_{x=-y^3+1}^{x=y^2+3y}$$

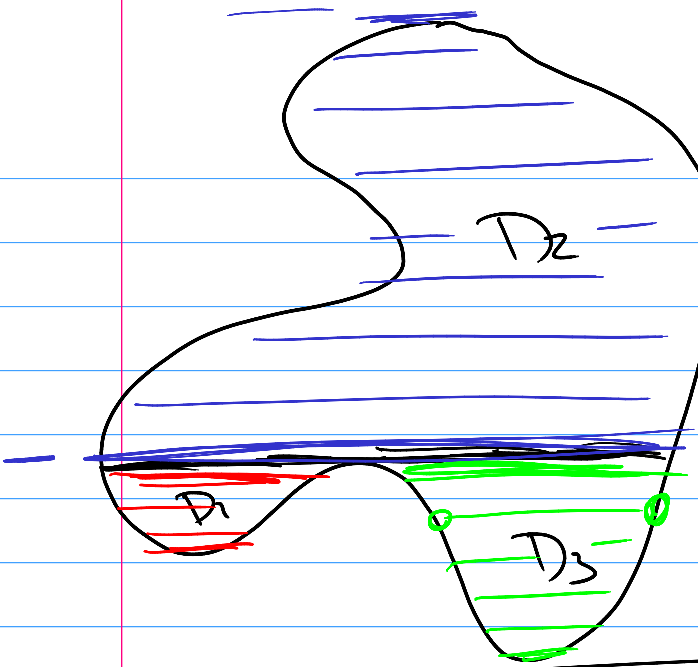
antideriv.

$$= \frac{-1}{y^2+3y} - \frac{-1}{-y^3+1}$$

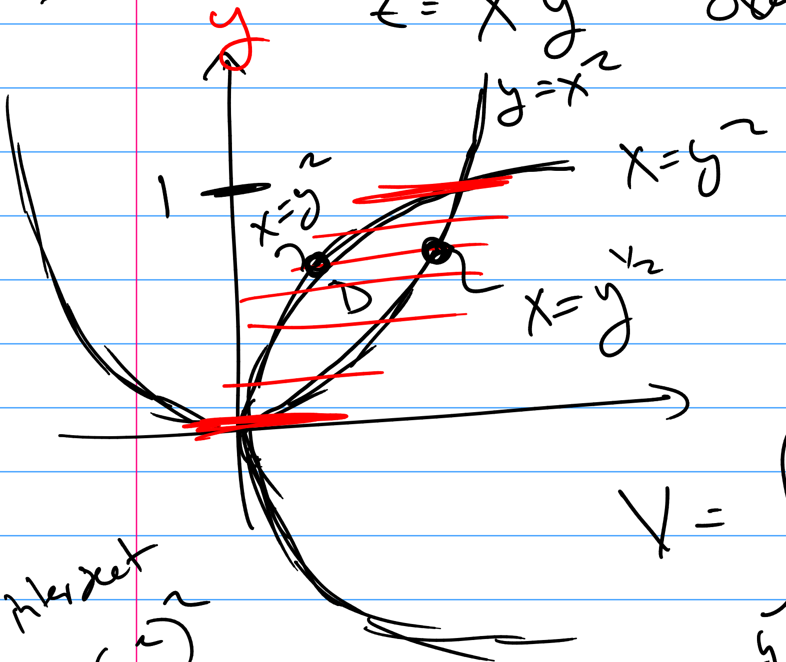
$$\iint_D f(x,y)\,dA = \iint_{D_1} f\,dA + \iint_{D_2} f\,dA + \iint_{D_3} f\,dA$$

weird D?





$z = x^2 y$ over D is between $y = x^2$ and



$$\iint x^2 y \, dA$$

$$V = \int_{y=0}^{y=1} \left[\int_{x=y^2}^{x=y^{1/2}} x^2 y \, dx \right] dy$$

intersect

$y = (y^2)^2$

$y = y^4 \quad y^4 - y = 0 \quad y(y^3 - 1) = 0$

New

$$\int_{y=0}^{y=1} \left[\int_{x=y^2}^{x=y^{1/2}} x^2 y \, dx \right] dy$$

$$= \int_{y=0}^{y=1} \left[\frac{1}{3} x^3 y \Big|_{x=y^2}^{x=y^{1/2}} \right] dy$$

$$= \int_{y=0}^{y=1} \left[\underline{\tfrac{1}{3}\left(y^{1/2}\right)^3 y - \tfrac{1}{3}\left(y^2\right)^3 y} \right] dy$$

= finish !