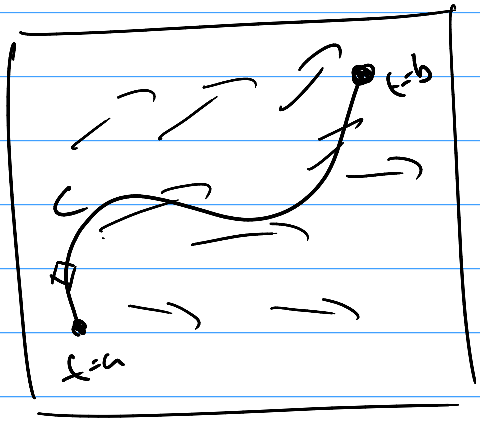
Math 3444

Calculus



$\mathbf{F} = \langle P(x,y), Q(x,y) \rangle$

Vector Field

$\boxed{16.2}$ Line Integral $\int_C f \, ds$ — scalar function

$\boxed{2D}$

have C + parametric $\mathbf{r}(t) = \langle x(t), y(t) \rangle$

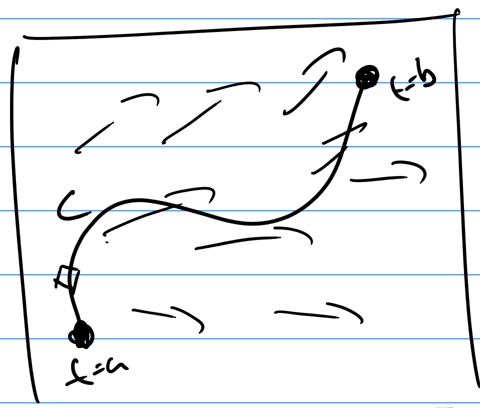
$a \le t \le b$

$\boxed{3D}$ $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle \quad a \le t \le b$

so $\boxed{\int_C f \, ds} = \int_a^b [f(\mathbf{r}(t)) |\mathbf{r}'(t)|] \, dt$

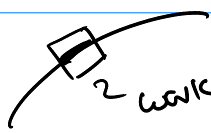
2D $= \int_a^b \left(f(x(t), y(t)) \sqrt{(x')^2 + (y')^2} \right) dt$

Application: $\boxed{\text{Work}}$



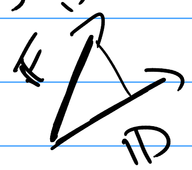
Work = (Force)(displacement)

?? add all the small "works" along C??

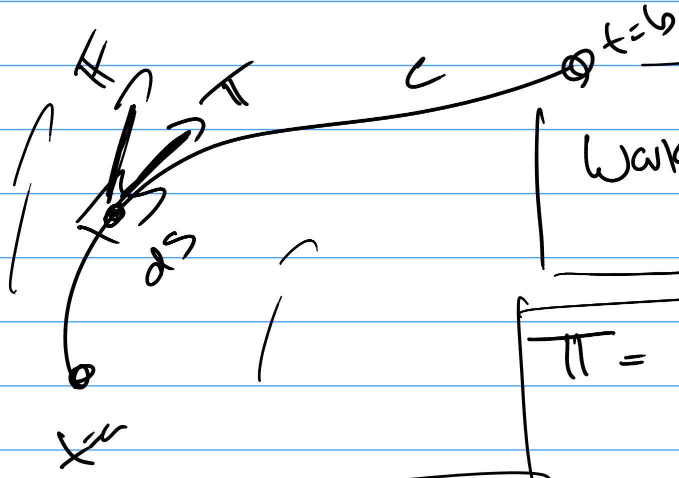


? work to move this small bit

Work = (force)(displacement)



Work = $\mathbb{F}\cdot\mathbb{D}$ scaler



$$\boxed{\text{Work} = \int_C (\mathbb{F}\cdot\mathbb{T})\,ds}$$

$$\boxed{\mathbb{T} = \frac{\mathbb{r}'}{|\mathbb{r}'|}} \qquad ds = |\mathbb{r}'|\,dt$$

C is $\boxed{\mathbb{r}(t) \quad a \le t \le b}$

$$\int_{t=a}^{t=b} \mathbb{F}\cdot\left(\frac{\mathbb{r}'}{|\mathbb{r}'|}\right)|\mathbb{r}'|\,dt$$

$$\boxed{\text{Work} = \int_C (\mathbb{F}\cdot\mathbb{T})\,ds} = \int_{t=a}^{t=b} (\mathbb{F}\cdot\mathbb{r}')\,dt$$

$$\boxed{\text{Work} = \int_{t=a}^{t=b} [\mathbb{F}(\mathbb{r}(t))]\cdot\mathbb{r}'(t)\,dt}$$

$\mathbb{r} = \langle x, y\rangle = \langle x(t), y(t)\rangle \qquad \mathbb{r}' = \langle x', y'\rangle$

$a \le t \le b$

Say 3D $\quad \mathbb{F} = \langle P(x,y,z), Q(x,y,z), R(x,y,z) \rangle$

C is $\quad \mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$

$a \le t \le b \quad \mathbf{r}'(t) = \langle x', y', z' \rangle$

Work $\displaystyle\int_C (\mathbb{F} \cdot \mathbb{T})\, ds$

$$\int_a^b \langle P, Q, R \rangle \cdot \langle x', y', z' \rangle \, dt$$

$$= \int_a^b \left(P\, \underbrace{x'}_{x'dt \,=\, dx} + Q\, \underbrace{y'}_{y'dt \,=\, dy} + R\, \underbrace{z'}_{z'dt \,=\, dz} \right) dt$$

$$= \int_a^b P\,dx + Q\,dy + R\,dz$$

$$= \int_a^b \langle P, Q, R \rangle \cdot \langle dx, dy, dz \rangle$$

$$= \int_C \mathbb{F} \cdot d\mathbf{r}$$

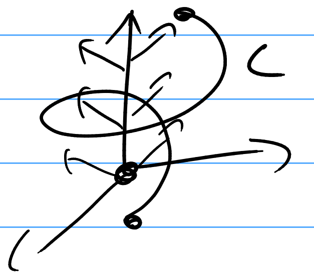
Notation for field

$$\boxed{\int_C \mathbb{F}\cdot d\mathbb{r}} = \boxed{\int_C (\mathbb{F}\cdot\mathbb{T})\,ds}$$

$$= \boxed{\int_a^b (\mathbb{F}(\mathbb{r}(t))\cdot\mathbb{r}')\,dt}$$

$$\int_C P\,dx + Q\,dy + R\,dz$$

$\boxed{ex}$ $\mathbb{F} = \langle y, x, z\rangle$



C is $\mathbb{r}(t) = \langle \cos(t), \sin(t), t\rangle$

$0 \le t \le 4\pi$

$$\boxed{\int_C \mathbb{F}\cdot d\mathbb{r}} = \boxed{\int_C [\mathbb{F}\cdot\mathbb{T}]\,ds}$$

$$= \boxed{\int_0^{4\pi} \underbrace{\langle \sin(t), \cos(t), z\rangle}_{\mathbb{F}(\mathbb{r}(t))}\cdot\underbrace{\langle -\sin(t), \cos(t), 1\rangle}_{\mathbb{r}'}\,dt}$$

$$= \int_0^{4\pi} \left(-\sin^2(t) + \cos^2(t) + 2\right) dt$$

$$= \dots =$$

Fundamental thm of Calculus

$$\int_a^b f \, dx$$

if $\frac{d}{dx}[F] = f$

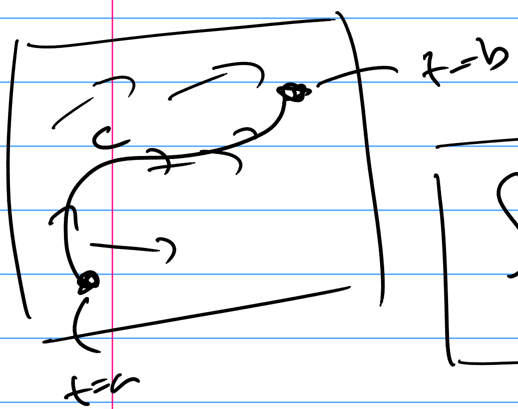
$$\Rightarrow \; = F(b) - F(a)$$

Fundamental thm of line integrals

Work: $\int_C \mathbf{F} \cdot d\mathbf{r}$ what if $\boxed{\mathbf{F} = \nabla f}$

C is $\mathbf{r}(t)$ $a \le t \le b$

$\mathbf{F}$ is a conservative vector field and f is called its potential function

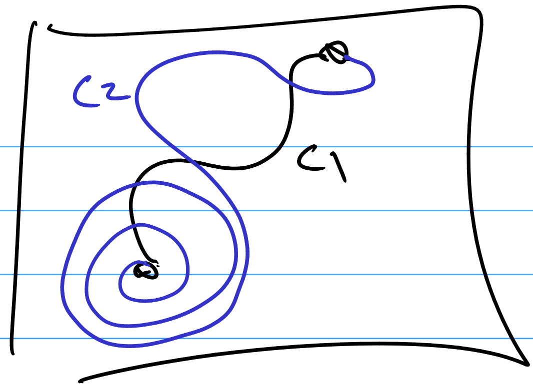


$$\boxed{\int_C \mathbf{F} \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))}$$

end point   start point

So if $\nabla f = \mathbb{F}$

$$\int_{C_1} \mathbb{F} \cdot dr = \int_{C_2} \mathbb{F} \cdot dr$$



$$= \boxed{f(\text{end}) - f(\text{start})}$$

$f = xyz$ ← $\nabla f = \langle yz, xz, xy \rangle = \mathbb{F}$

r

$$\int_C \langle yz, xz, xy \rangle \cdot dr$$

$=$
