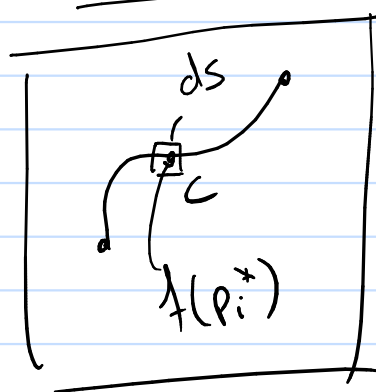
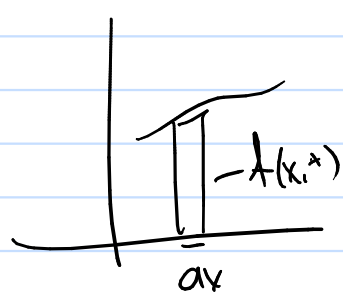
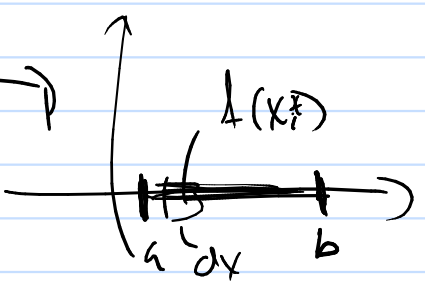


Math 344

$$\int_a^b f(x) dx$$



$$ds = |r'| dt$$

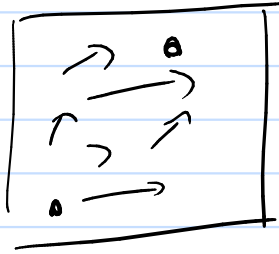
$$\int_C f ds$$

$$r(t) = \langle x(t), y(t), z(t) \rangle$$

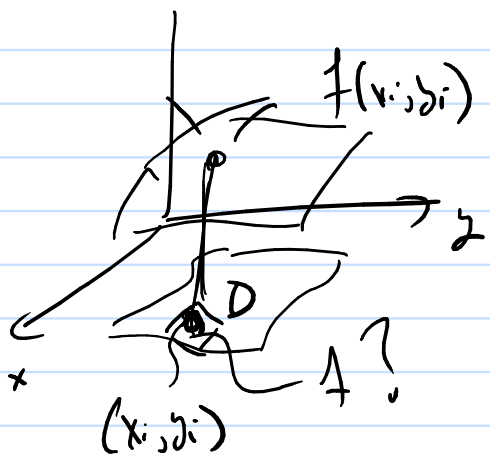
$$a \leq t \leq b$$

Applications

$$\int_C f(x, y, z) ds$$



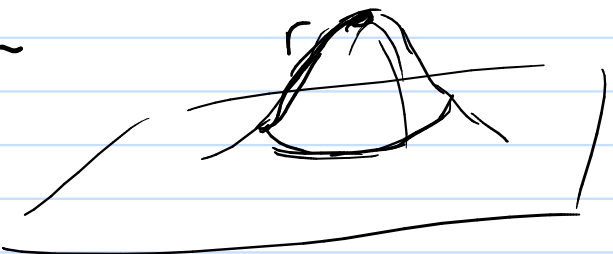
What about ...



$$\iint_D f(x, y, z) dA$$

$$\pi r^2$$

$$\text{Area} = \pi r^2$$



$y = x^2$
 $x =$
 r

$\int_0^1 2\pi r \, dy$
 $\int_0^1 2\pi \sqrt{y} \, dy$

Arch length?
Arch surface area?

$\iint_D 1 \, dA$

what happens

$P_i = (x_i, y_i)$
 $\circ P_i, f(x_i, y_i)$

D

Parametric Curves

$C \subset \mathbb{D}$
 $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$
 $a \leq t \leq b$

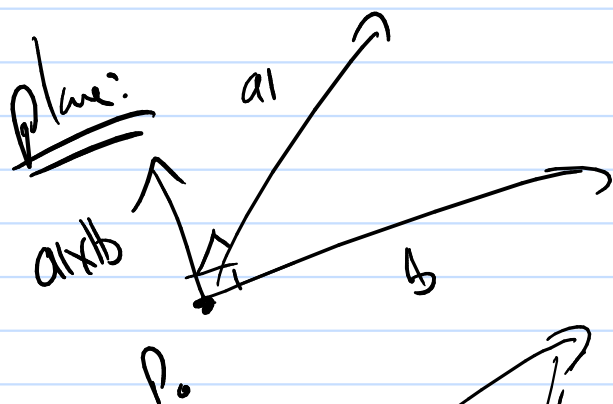
Parametric Surfaces

$\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$

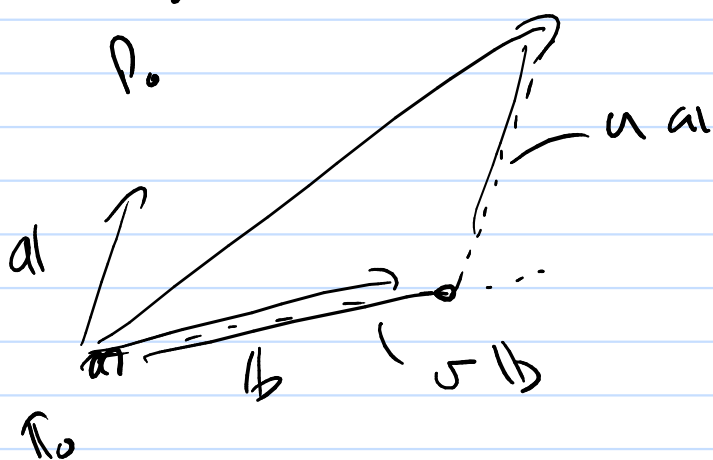
u is const v , v are in \mathbb{D}
 u varies a flat 2D domain

given $\pi \rightarrow$ plot parametriz surface (use tech.)

surface? \rightarrow find π ?



plane needs P_0 & point
and a normal $a \times b$



any $\pi = P_0 + u \cdot a + v \cdot b$

$$\textcircled{\infty} \quad a = \langle 1, 1, 1 \rangle \quad b = \langle 0, 1, 0 \rangle$$

through $\langle 0, 0, 0 \rangle = P_0$

$$\pi = \langle 0, 0, 0 \rangle + u \langle 1, 1, 1 \rangle + v \langle 0, 1, 0 \rangle$$

$$\pi = \langle u, u+v, u \rangle$$

any surface

$$z = f(x, y)$$

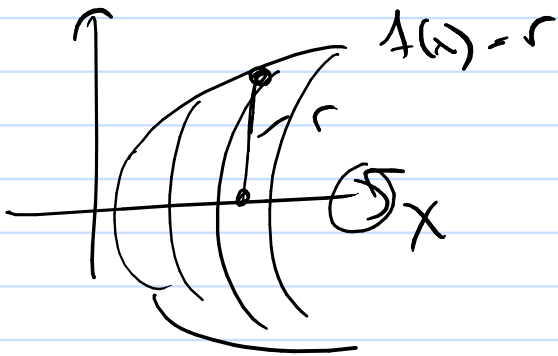
use

$$x = u$$

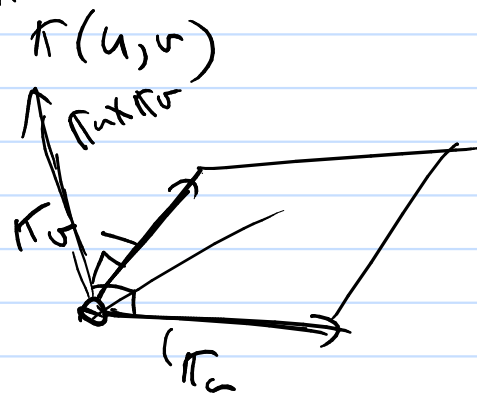
$$y = v$$

$$z = f(u, v)$$

Surface of revolution



$$\begin{aligned} x &= u \\ y &= f(u) \cos v \\ z &= f(u) \sin v \end{aligned}$$



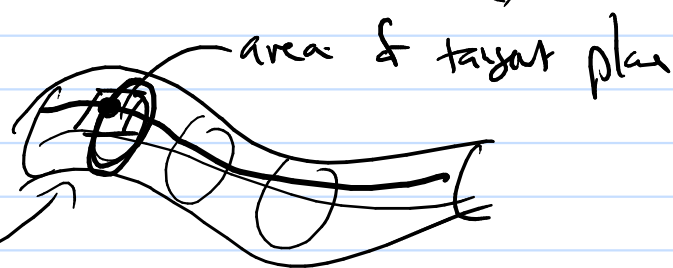
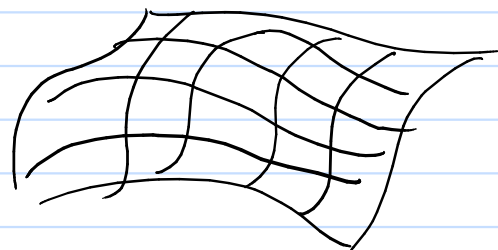
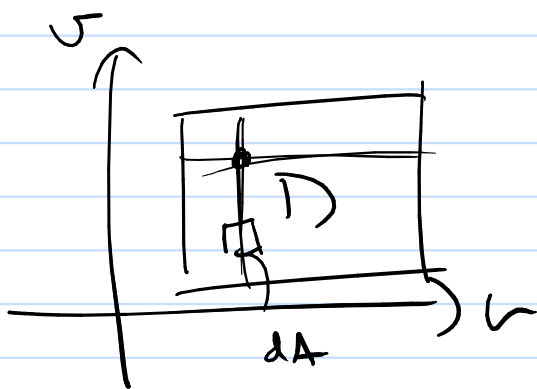
Note: if $n_u \wedge n_v \neq \mathbf{0}$

all S smooth
(no corners)

Find Area of our parametrized Surface

$$\mathbf{r}(u, v) = \langle x(u, v), y(u, v), z(u, v) \rangle$$

u, v are from D .



$$\iint_D |\mathbf{r}_u \times \mathbf{r}_v| dA$$

$$\underline{\underline{|\mathbf{r}_u \times \mathbf{r}_v|}} = \text{area of tangent plane}$$