Math 344

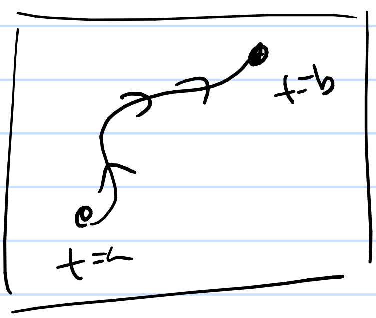
$\boxed{\text{Ch 16}}$ Skills to have

① $\int_a^b f\,dx$, $\iint_D f\,dA$, $\iiint_E f\,dV$

② parametric curves $\mathbf{r}(t)$ $a \le t \le b$

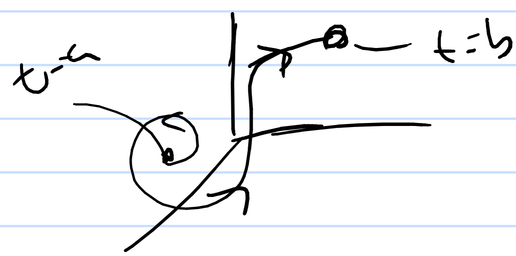
2D

$\mathbf{r}(t) = \langle x(t), y(t) \rangle$

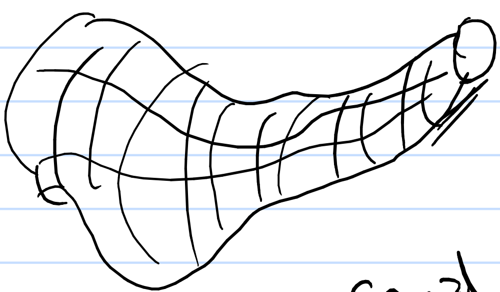


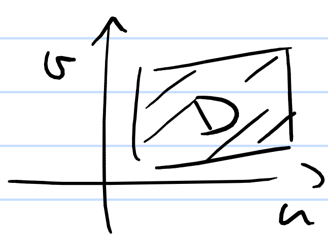
3D

$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$



③ parametric surface $\mathbf{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$





Visualize!

Special case:

$z = f(x,y)$ over D

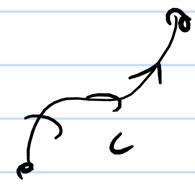
$\mathbf{r}(u,v) = \langle u, v, f(u,v) \rangle$

$u, v \in D$

16.1 Visualize $\mathbb{F}$, Vector Fields

Making your own study sheet.

16.2 Line Integrals



C is $r(t)$

$$\int_C f\,ds = \int_a^b f(r(t))\,|r'(t)|\,dt$$

how?

todo?

(1) 2D example

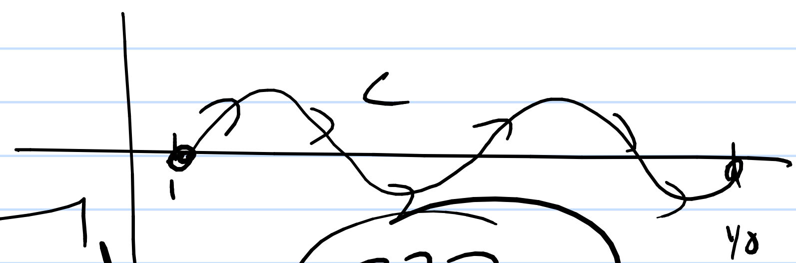
$r(t) = \langle 3t+1, \sin(t) \rangle$

$0 \le t < 4\pi$

$r'(t) = \langle 3, \cos t \rangle$

$|r'(t)| = \sqrt{3^2 + \cos^2 t}$

$$\int_C xy\,ds$$



$$= \int_0^{4\pi} (3t+1)\sin t \sqrt{9+\cos^2 t}\,dt = ???$$

(2) 3D?

$r(t) = \langle 3t+1, \sin t, \cos t \rangle$

$0 \le t \le 4\pi$

16.2 $\displaystyle\int_C \mathbb{F}\cdot d\mathbb{r} = \int_C (\mathbb{F}\cdot\mathbb{T})\,ds$ "work"

$$= \int_a^b \mathbb{F}(\mathbb{r}(t))\cdot\mathbb{r}'(t)\,dt$$

how?

to do? (1) 2D?

(2) 3D?

$$\mathbb{r}'(t) = \langle 2, -\sin t, \cos t\rangle$$

C: $\boxed{\mathbb{r}(t) = \langle 2t, \cos t, \sin t\rangle \quad 0 \le t \le 4\pi}$

$\displaystyle\int_C \mathbb{F}\cdot d\mathbb{r}$ if $\mathbb{F} = \langle -x, -z, y\rangle$

$$= \int_0^{4\pi} \underbrace{\langle -2t, -\sin t, \cos t\rangle}_{\mathbb{F}} \cdot \underbrace{\langle 2, -\sin t, \cos t\rangle}_{\mathbb{r}'}\,dt$$

$$= \int_0^{4\pi} \left(-4t + \sin^2 t + \cos^2 t\right) dt = \int_0^{4\pi} (1-4t)\,dt$$

$$= ??$$

16.3 / 16.4 special cases for $\int_C \mathbb{F} \cdot d\mathbb{r} = \int_C (\mathbb{F} \cdot \mathbb{T})\, ds$

16.3 $\int_C (\nabla f) \cdot d\mathbb{r} = f(\mathbb{r}(b)) - f(\mathbb{r}(a))$

todo?

① check if $\mathbb{F} = \nabla f$

how? $\mathbb{F} = \langle P, Q \rangle$ if $P_y = Q_x$

2D

② if $\langle P, Q \rangle = \nabla f = \langle f_x, f_y \rangle$

solve $\begin{matrix} f_x = P \\ f_y = Q \end{matrix} \to f$

ex $f = x^2 y + y^3$ $\quad f_x = 2xy$

$f_y = x^2 + 3y^2$

$\mathbb{F} = \langle 2xy, x^2 + 3y^2 \rangle = \langle P, Q \rangle$

is it conserv? check P_y, Q_x

$P_y = 2x$, $Q_x = 2x$ yep!

so $\begin{matrix} f_x = 2xy \\ f_y = x^2 + 3y^2 \end{matrix} \xrightarrow{?} f = ???$

todo (3) $\int_C (\nabla f)\cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$

↑ use your found f

ex $\int_C \langle 2xy, x^2+3y^2\rangle \cdot d\mathbf{r}$ $\quad$ C is $\langle 2t, \cos(t)\rangle$

$0 \le t \le \pi/2$

found $f = x^2y + y^3$

$\mathbf{r}(\pi/2) = \langle \pi, 0\rangle$

$\mathbf{r}(0) = \langle 0, 1\rangle$

$x \quad y$

So $= f(\pi,0) - f(0,1)$

$= ???$

16.4 Greens

2D

$\oint_C \mathbf{F}\cdot d\mathbf{r}$

$\mathbf{F} = \langle P, Q\rangle$

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \iint_D (Q_x - P_y)\,dA$$

todo (1) $\oint_C \mathbf{F}\cdot d\mathbf{r} \longrightarrow \iint_D (Q_x - P_y)\,dA$

$= ???$

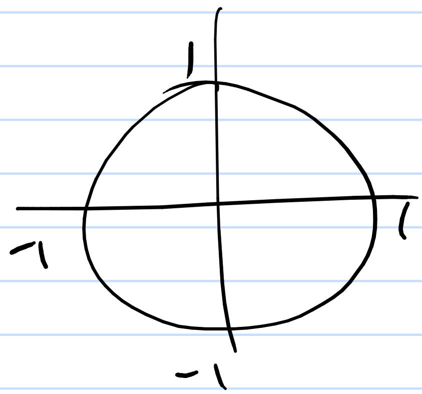
ex $\mathbf{F} = \langle 3xy, x^2+y^2\rangle$

C is the unit circle around the origin

$\mathbb{F} = \langle 3xy, x^2+y^2 \rangle$

$$\oint_C \mathbb{F}\cdot d\mathbf{r} = \iint_D (2x - 3x)\, dA$$

$$= -\iint_D x\, dA$$



$\boxed{16.5}$ $\text{curl}(\mathbb{F}) = \nabla \times \mathbb{F}$

$\text{div}(\mathbb{F}) = \nabla \cdot \mathbb{F}$

howto? do <u>several</u> $\nabla \times \mathbb{F}$, $\nabla \cdot \mathbb{F}$

<u>16.6</u> toolbox idea & $\mathbf{r}(u,v)$

$\boxed{16.7 \text{ to } 16.9}$ $\iint_S 1\, dS$