Math 530

show N(E)

5.1 (1, 3, 7, 11, 15, 17)

5.2 (1, 3, 5, 7, 9, 15, 29)

If possible, give more than one verbal reasoning.

AI 5c

Set theory

	Men	Women
Married	S_1: 5	S_2: 8
unmarried	S_3: 2	S_4: 5

4 + people

$|S_1 \cup S_3| + |S_2 \cup S_4|$

$||S_1 \cup S_2 \cup S_3 \cup S_4|| = 20$

$|S_2 \cup S_4| = 13$

$|S_1 \cup S_3| = 7$ ✓

$|S_2| = 8$

$|S_4| = 5$

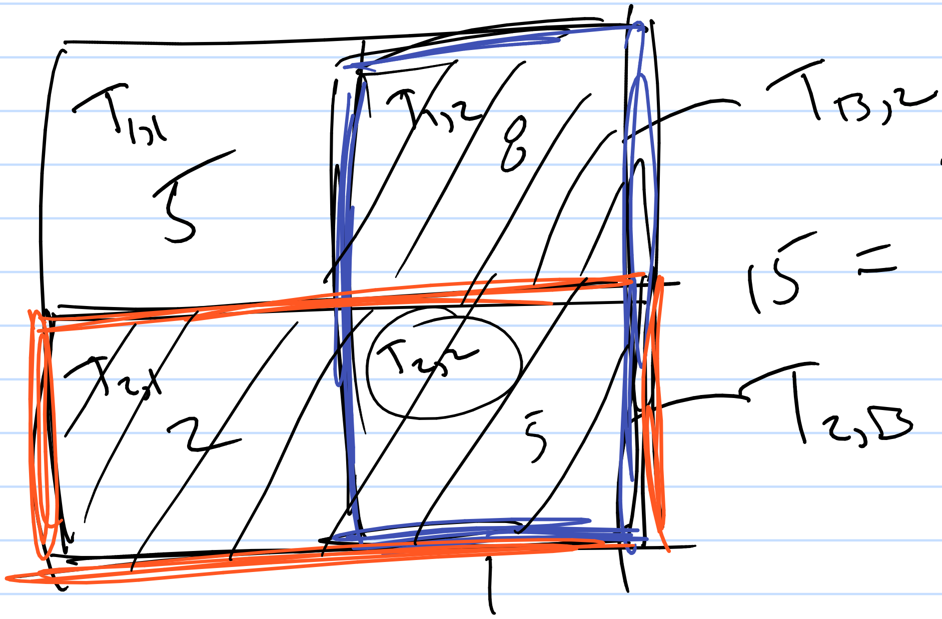
5c) |Women or unmarried| = 15

$\underbrace{S_2 \cup S_4}_{\text{women}} \cup S_3$, with $S_4 \cup S_3$ unmarried

$|S_3 \cup S_4| = 15$ so $|S_3| = 10$

$\underbrace{(S_2 \cup S_4)}_{13} \cup \underbrace{(S_3 \cup S_4)}_{7} = \underline{15}$

ans?



$$15 = |T_{13,2} \cup T_{2,13}|$$

Proof by induction: — well ordered set of objects (2 parts)

part #1 | Basis Step — show true for smallest element

Part #2 | Inductive Step — assume true for 1^{st} to K^{th}; using those show $K+1^{st}$ is also true.

for $n = \boxed{0}, 1, 2, \ldots$ $\quad 1+3+5+\cdots+(2n+1) = (n+1)^2$

Basis: $(n=0)$ $\quad 1 = (1)^2$ true

Inductive: assume

$$1 = (1)^2$$
$$1+3 = (2)^2$$
$$1+3+5 = (3)^2$$
$$\vdots$$
$$1+3+5+\cdots+(2K+1) = (K+1)^2$$

show?

$$1+3+\cdots+(2K+1)+(2K+3) \overset{?}{=} (K+2)^2$$

$$\underbrace{1+2+3+\dots+(2k+1)}_{I}+(2k+3)$$

$$= (k+1)^2 + (2k+3)$$

$$= (k+1)^2 + (2k+3)$$

$$= k^2+2k+1+2k+3$$

$$= k^2+4k+4$$

$$= (k+2)^2$$

goal?:

$$\dots = (k+2)^2$$

k^2+4k+4

Counting

Sum Rule / Inclusion-Exclusion

$$|S_1 \cup S_2| = |S_1| + |S_2| - |S_1 \cap S_2|$$

Product/Mult. Rule $\rightsquigarrow$ Division Rule

$$|S_1 \times S_2| = |S_1||S_2|$$

$$|S_1 \times S_2| = \frac{|S_1||S_2|}{d}$$

d ← ways of non-uniq comb.

Combinations / Permutations.

arrange n uniq. objects (w/o repeats)

$$n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot 2 \cdot 1 = n!$$

1st $\wedge$ 2nd $\wedge$ 3rd

and and

Permutations: also called n-permutation.

r picked from n same as ignore $(n-r)$ objects

$$\underbrace{O_1\; O_2\; O_3 \cdots O_r}_{r \text{ objects (order matters)}}\;\Big|\;\underbrace{O_{r+1}\; O_{r+2} \cdots O_n}_{n-r \text{ objects (don't care)}}$$

$$\boxed{n(n-1)(n-2)\cdots(n-(r-1))} \leftarrow \text{ans.}$$

$7! = 7 \cdot 6 \cdot 5!$

idea

$$\frac{n!}{(n-r)!} = \frac{n(n-1)(n-2)\cdots(n-(r-1))\,\cancel{(n-r)!}}{\cancel{(n-r)!}}$$

$$\boxed{P(n,r) = \frac{n!}{(n-r)!} = n(n-1)(n-2)\cdots(n-(r-1))}$$

choose

from n choose r

$$\underbrace{O_1\, O_2\, \cdots\, O_r}_{r \text{ objects}} \;\Big|\; \underbrace{O_{r+1}\, O_{r+2}\, \cdots\, O_n}_{n-r \text{ objects}}$$

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

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n objects partition into $r_1, r_2, r_3, \dots, r_m$

$$|r_1| + |r_2| + \cdots + |r_m| = n$$

$$\underbrace{\binom{n}{r_1} \cdot \binom{n-r_1}{r_2} \cdot \binom{n-r_1-r_2}{r_3} \cdots}_{} = \underbrace{\frac{n!}{r_1!\, r_2! \cdots r_m!}}_{}$$

$$= P(n; r_1, r_2, \dots, r_m)$$
