Math 530

Due Sept 8 — 5.3 (1, 3, 5, 7, 9, 13)

5.4 (1, 5, 7, 11, 13, 19, 21)

Review

5.1 "or" Addition Principle / Inclusion-Exclusion principle

Overcount

"and" Mult. Principle / Division Principle → $\frac{\text{total ways}}{d}$

↳ d arrangements of non-distinct Composite outcomes

distinct

5.2 Arrangement of r objects w/o replacement

ways $= r!$

Applets

Pick r from n with order (permutation) $P(n,r) = \frac{n!}{(n-r)!}$

Select w/o order

Choose r from n

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Note: $\binom{n}{r} = \binom{n}{n-r}$

5.3 Arrangements with repetitions (identical elements)

n objects move into box$_1$ of r_1
box$_2$ of r_2
⋮
box$_m$ of r_m

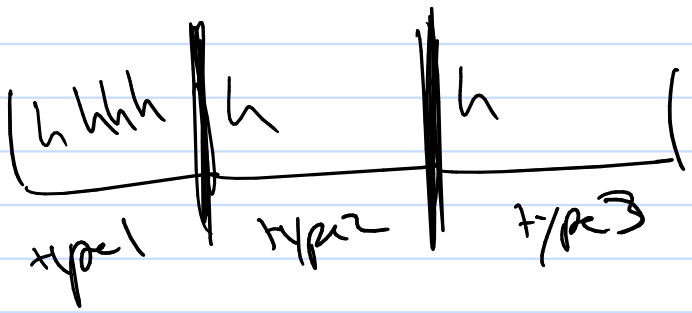
$$\binom{n}{r_1} \cdot \binom{n-r_1}{r_2} \cdot \binom{n-r_1-r_2}{r_3} \cdots \binom{n-r_1-\cdots-r_{m-1}}{r_m}$$

$$\frac{n!}{r_1!\,(n-r_1)!} \cdot \frac{(n-r_1)!}{r_2!\,(n-r_1-r_2)!}$$

$$= \frac{n!}{r_1!\,r_2!\cdots r_m!} = P(n; r_1, r_2 \ldots, r_m)$$

5.3 Select r things of K possible types.

cf book → Select 6 hot dogs & 3 types



2 symbols |, h

8 blanks

h h h h | h | h

$$\binom{8}{6} = \frac{8!}{6!\,2!}$$

Select r of K types

$$\binom{r+K-1}{r} = \frac{(r+K-1)!}{r!\,(K-1)!}$$

(ex) pile of 1¢, 5¢, 10¢, 25¢ coins

how many ways to take 10?

it is a select 10 of 4 types.

$$C((10+4-1), 10) = C(13,10) = \frac{13!}{10!\,3!}$$

Q's Appendix 4 #4

10-F, 20-S, 8-G, 15-R, 25-I guarantee

ans? 52

$\left\lceil \frac{N}{5} \right\rceil$ = n objects in at least one box

F S G R I

$N = 56$

$\left\lceil \frac{N}{5} \right\rceil = 12$

$\left\lceil \frac{11-5+1}{5} \right\rceil$ $\left\lceil 11+\frac{1}{5} \right\rceil$ = 12

11, 11, 11, 11, 11

10, 11, 8, 11, 11 $+1$ = 52

5.4 Distributions (of stuff into boxes)

① Distribute distinct objects → arrangement problem

② Distribute non-distinct objects → selection problem of k types

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distinct Distribute r distinct objects into n places. How many?

→ Consider this to be a string of length r of n symbols.

o_1 o_2 ... o_r → $s_1 s_2 \cdots s_n$ → $n \cdot n \cdots n$ (r times)

Ways = n^r distributions

Distribute r distinct objects into n places $= n^r$

Distribute with the constraint of r_1 into place 1
r_2 into place 2
$\vdots$
r_n into place n

$$P(r; r_1, r_2, \ldots, r_n) = \frac{r!}{r_1! r_2! \cdots r_n!} \text{ distributes}$$

(ex) 15 people and 3 rooms to go into

total distrib. $= 3^{15}$

(ex) 15 people want 5 in room 1, 7 in room 2
3 in room 3

distributes $= \frac{15!}{5! 7! 3!} = P(15; 5, 7, 3)$

Non-unique distrib. r into n boxes

$$C(r+n-1, r) = \frac{(r+n-1)!}{r!\,(n-1)!}$$