Math 530

Q's — 5.4 #7

V, I, S, I, T, I, N, G

$G, N, T, S, V, I_1, I_2, I_3$

$\boxed{C(8,5)\cdot 5!}$

arrangements

thought #1 — total (uniq) $\boxed{8!}$

thought #2 — Notice that $I_1 = I_2 = I_3$

$\boxed{\frac{8!}{3!}} = P(8,5)$

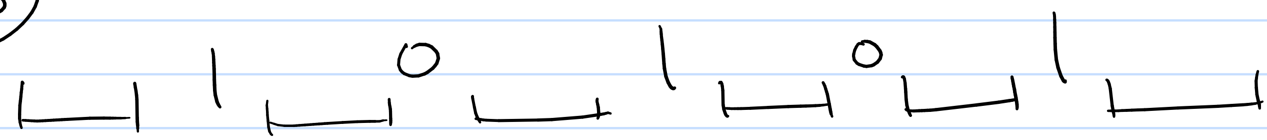
Obs: no consec. I's?

1, 1, 1, 0, 0, 0, 0, 0, 0

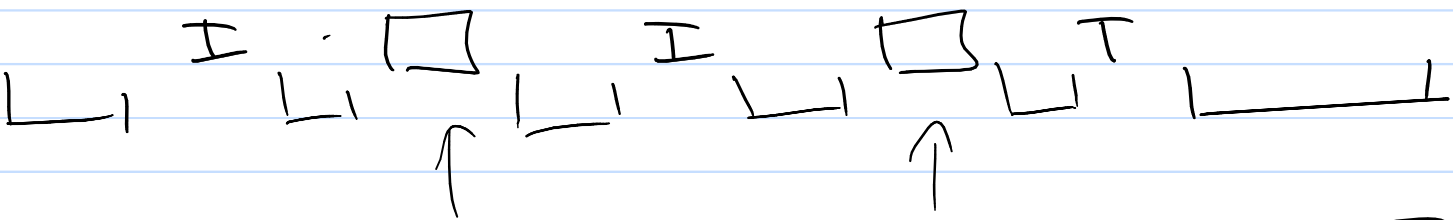
$$\boxed{\binom{(5-2)+4-1}{5-2}(5!)}$$

$$C(n+r-1, r) = \binom{n+r-1}{r}$$

(10)



I, I, I, V, S, T, N, G



$$C\binom{(5-2)+4-1}{(5-2)}5! = \boxed{\binom{6}{3}5!}$$

Note: #1 today 5.5, next two classes are 6.1, 6.2

5.5 (1, 2b, 5, 14ab)

5.5 binomial th^m

$$(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \dots + \binom{n}{n} a^0 b^n$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Background: to show expression #1 = expression #2

tech #1 expression #1 = (step #1)

= (step #2)

⋮

= expression #2

tech #2 Start with a known equality.

(known #1 = known #2) is true

logically correct equation steps

step #1 (new #1 = new #2)

(expression #1 = expression #2)

to show expression #1 = expression #2

by counting / combinatorial proof is ...

ex

task to count

① count it and find an expression #1

=

② count it another way and find expression #2

ex

have 5 winners, in order, from 8

tech #1 $P(8,5)$

$\quad\quad\; \parallel$

tech #2 $C(8,5) \cdot 5!$

$$P(8,5) = C(8,5)\, 5!$$

Algebra: $\dfrac{8!}{3!} = \dfrac{8!\,\cancel{5!}}{\cancel{5!}\,3!}$

$$(a+b)^n = \underbrace{(a+b)\cdot(a+b)\cdot(a+b)\cdot \; \cdots \; \cdot(a+b)}_{n\text{-factors}}$$

$$= \frac{n!}{n!\,0!} a^n b^0 + \frac{n!}{(n-1)!\,1!} a^{n-1} b^1 + \frac{n!}{(n-2)!\,2!} a^{n-2} b^2 + \cdots \quad + \frac{n!}{0!\,n!} a^0 b^n$$

$$\boxed{(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \cdots \quad + \binom{n}{n} a^0 b^n}$$

Note: why is $0! = 1$ 00000

$$c(5,5) = \frac{\cancel{5!}}{\cancel{5!}\,0!} = 1$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$= \frac{1}{0!} = 1$$

Use the equality. $(a+b)^n = \binom{n}{0} a^n b^0 + \binom{n}{1} a^{n-1} b^1 + \cdots + \binom{n}{n} a^0 b^n$

at $a=1, b=1$

$$2^n = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n}$$

$a=1 \quad b=-1$

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \cdots$$

$$0 = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + -$$

$$\binom{n}{1} + \binom{n}{3} + \cdots = \binom{n}{0} + \binom{n}{2} + \cdots = \frac{1}{2}2^n = 2^{n-1}$$

Pascal's Identity $\boxed{\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}}$ book's

Counting proof: of n people, Mark is in the group

so $(n-1)$ + Mark $= n$

task

from everyone choose k to get snacks.

Ver #1 $\binom{n}{k}$

Ver #2 disjoint sets are

(1) Mark doesn't get snacks $\binom{n-1}{k}$

or

(2) Mark gets snacks $1 \cdot \binom{n-1}{k-1}$

$$\boxed{\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}}$$

Pascal's triangle

$(x+y)^5$

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

n=0 1

n=1 1 1

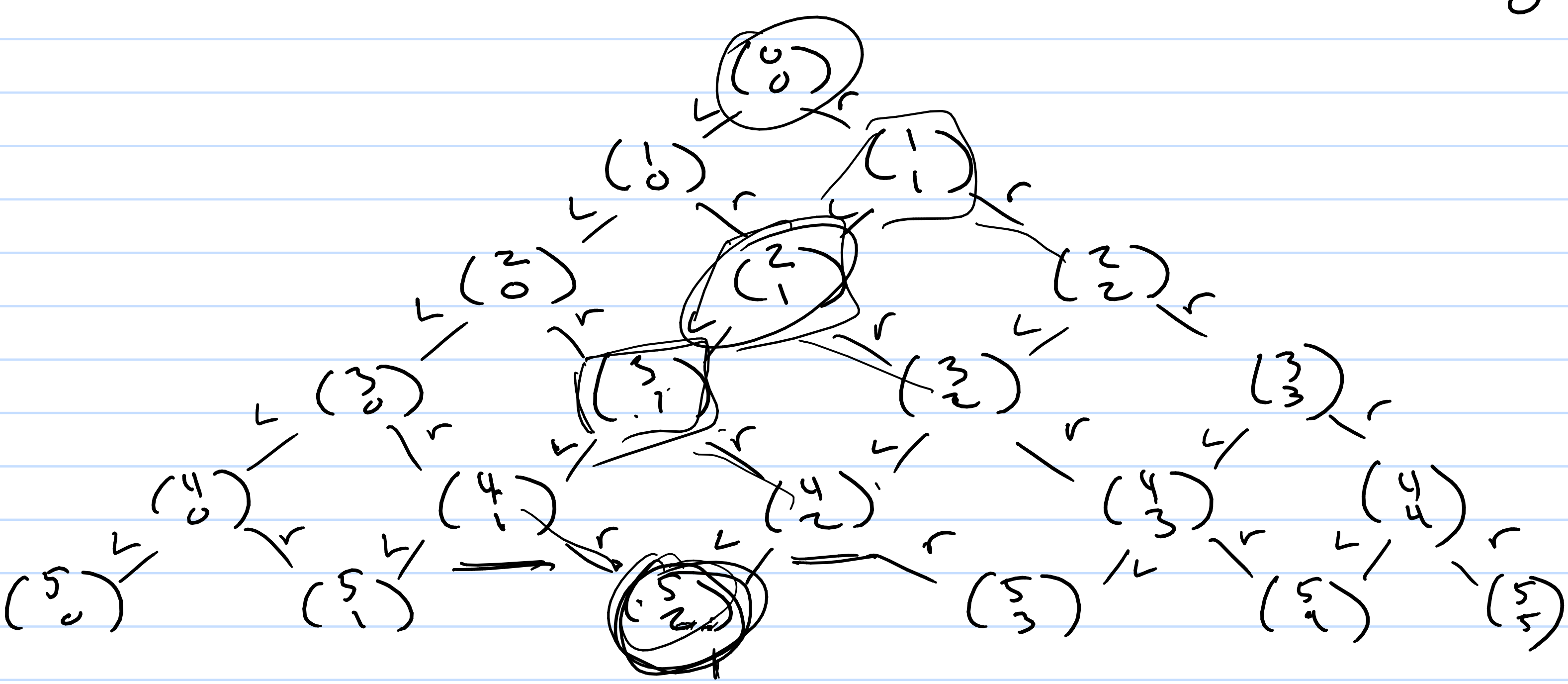
n=2 1 2 1

1 3 3 1

1 4 6 4 1

→ 1 5 10 10 5 1

$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$$



$s\binom{n}{r}$ = block counting

$s\binom{n}{r} = c(n,r)$

$c(5,2) = c(4,1) + c(4,2)$

$$\boxed{C(n,r) = C(n-1,r-1) + C(n-1,r)}$$