Math 530

6.1 "fix"

6.2

$$\boxed{(1+x^1)^n}$$

take "power" of type.

polynomial = type $\left(\begin{smallmatrix}1\\ \text{or}\\ 0\end{smallmatrix} x^0 + \begin{smallmatrix}1\\ \text{or}\\ 0\end{smallmatrix} x^1 + \begin{smallmatrix}1\\ \text{or}\\ 0\end{smallmatrix} x^2 + \cdots\right)$

T = 1 = possible

F = 0 = not possible

ex even amounts $(x^2 + x^4 + x^6 + \cdots)$

or

0 is even?

$(x^0 + x^2 + x^4 + \cdots)$

$(0x^0 + 0x^1 + 1\cdot x^2 + 0\cdot x^3 + 1\cdot x^4 + \cdots)$

$(x^0 + x^1)^n$

$$(x^0+x^1)^3 = \underbrace{(x^0+x^1)}_{\text{type}}\underbrace{(x^0+x^1)}_{\text{type}}\underbrace{(x^0+x^1)}_{\text{type}}$$

$= 1x^0 + 3x^1 + \boxed{3}x^2 + 1\cdot x^3$

$= \binom{3}{0}x^0 + \binom{3}{1}x^1 + \binom{3}{2}x^2 + \binom{3}{3}x^3$

$= 1x^0$

types of 0 to ∞ Poly $(x^0 + x^1 + x^2 + \cdots)$

3 types of 0 to ∞ each

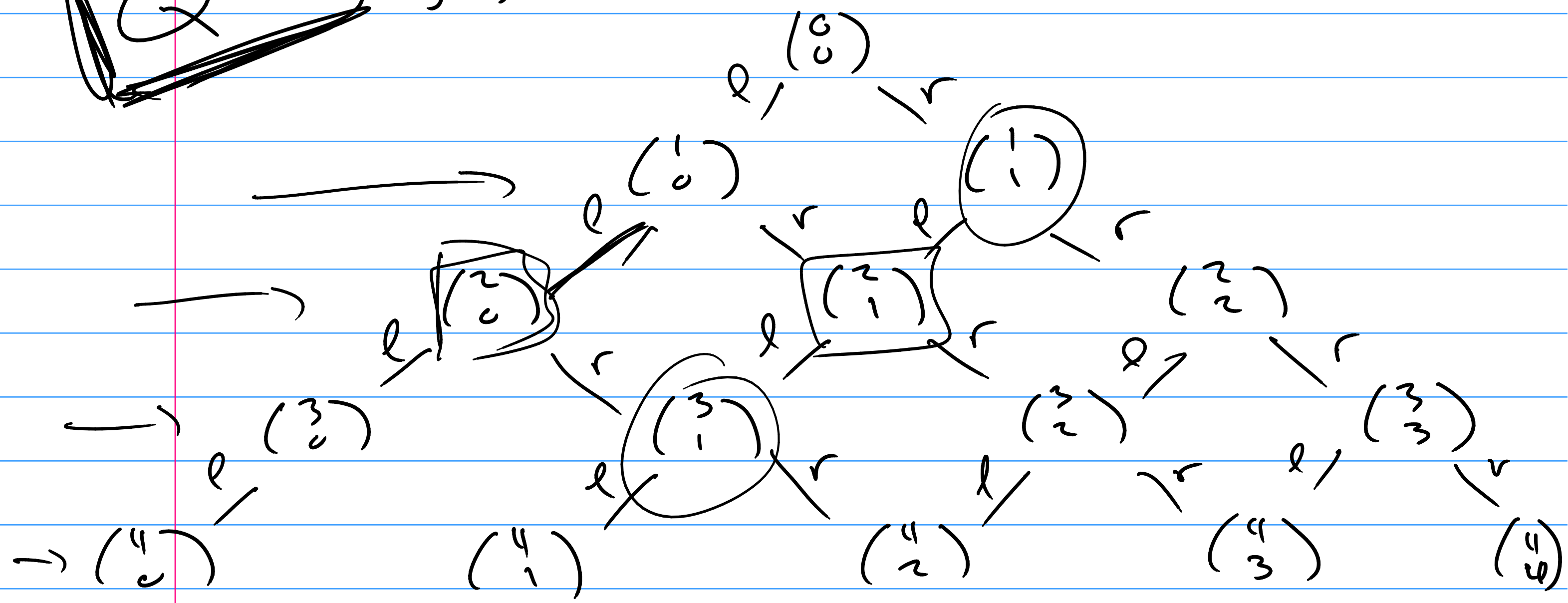
$$\underbrace{(x^0 + x^1 + x^2 + \cdots)^3}$$

$$= \sum_{r=0}^{\infty} \binom{r+3-1}{r} x^r$$

$$x^{e_1} + x^{e_2} + e^{e_3} \qquad e_1 + e_2 + e_3 = r$$

$$0 \le e_i$$

Q's 5.5 #26



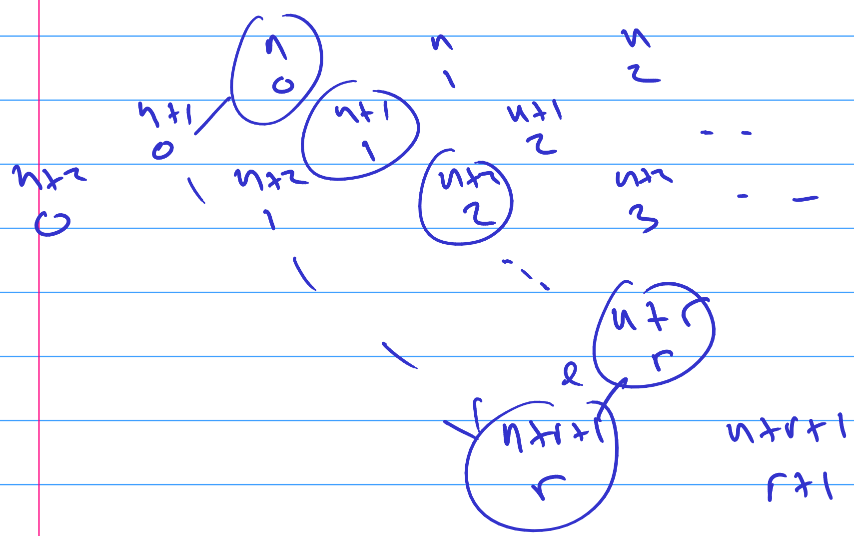
$$\binom{2}{0}\cdot 1 + \binom{2}{1}\cdot 1 = \binom{3}{1}$$

(7) $\binom{n}{0}+\binom{n+1}{1}+\binom{n+2}{2}+\cdots+\binom{n+r}{r}=\binom{n+r+1}{r}$

(*) $n=2 \quad r=2$

$$\binom{2}{0}+\binom{3}{1}+\binom{4}{2}=\binom{2+2+1}{2}$$

$$\binom{0}{0}$$
$$\binom{1}{0} \quad \binom{1}{1}$$
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$
$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$
$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$
$$\binom{5}{0} \quad \binom{5}{1} \quad \binom{5}{2} \quad \binom{5}{3} \quad \binom{5}{4} \quad \binom{5}{5}$$



5.5 (14b) $(1+x)^n=\sum_{k=0}^{n}\binom{n}{k}x^k$

? 1) 0

$$=\boxed{\sum_{k=0}^{n}k(k-1)\binom{n}{k}} \quad ? \; 0$$

So $\frac{d}{dx}(1+x)^n=\frac{d}{dx}\left[\sum_{k=0}^{n}\binom{n}{k}x^k\right]$

$$=\frac{d}{dx}\left[1+\binom{n}{1}x+\binom{n}{2}x^2+\binom{n}{3}x^3+\cdots\right]$$

2^{nd} deriv

$$= \binom{n}{1} + 2\binom{n}{2}x + 3\binom{n}{3}x^2 + \cdots$$
$$\cdots + k\binom{n}{k}x^{k-1}$$

$$2(1)\binom{n}{2} + 3(2)\binom{n}{3}x + \cdots + k(k-1)\binom{n}{k}x^{k-2}$$

So $\sum_{k=0}^{n} k(k-1)\binom{n}{k}$ is for $x=1$

So 2^{nd} deriv @ $x=1$

So $(1+x)^n \rightsquigarrow$ 1^{st} deriv $n(1+x)^{n-1}$

2^{nd} deriv $n(n-1)(1+x)^{n-2}$

etc ...

HW 6.1

(1) ways $\quad (x^0+x^1+x^3)(x^0+x^1+x^3)(x^0+x^1)(x^0+x^1)$

(a) to get 11 $\quad x^{11}$

$x^1x^1x^1x^1$

$x^3x^1x^0x^0$

(3) 5 red, 5 black, 4 white select r

3 types

$$(x^0+x^1+x^2+\cdots+x^5)(x^0+x^1+\cdots+x^5)(x^0+x^1+\cdots+x^4)$$

$$(x^0+x^1+\cdots+x^5)^2(x^0+x^1+\cdots+x^4)$$

6.2 #3 $a_7 x^7$?

$$(x^0+x^2+x^4)(x^0+x^1)^n$$

$a_7 x^7$

$$[x^0 + x^2 + x^4]\left(\sum_{k=0}^{n} \binom{n}{k} x^k\right)$$

$$x^0\binom{n}{7}x^7 + x^2\binom{n}{5}x^5 + x^4\binom{n}{3}x^3$$

$$a_7 = \binom{n}{7} + \binom{n}{5} + \binom{n}{3}$$

$$a_7 = \frac{n!}{7!\,(n-7)!} + \frac{n!}{5!\,(n-5)!} + \frac{n!}{3!\,(n-3)!}$$

Exam 1 5.1 – 5.5 / 6.1 – 6.2

12 probs @ 10pts
110pts = 100%

5.1 2 probs Sum / prod / inclusion-exclusion / div.

5.2 2 probs $C(n,r)$ $P(n,r)$

5.3 2 probs $P(n; r_1, r_2, r_3, \ldots, r_k)$

$C(n+r-1, r)$

5.1 $\boxed{\text{2 probs}}$

① select r of n types

$\equiv$

② distrib. r into n boxes

$\equiv$

④ $x_1 + x_2 + \dots + x_n = r$

$0 \le x_i$

5.5 $\boxed{\text{2 probs}}$

① use thm to make equality

② prove binomial thm by induction

$\boxed{\begin{matrix}6.1\\6.2\end{matrix}}$ 2 probs: ① word prob $\rightleftarrows$ prod of poly

② find a_r of $a_r x^r$