Math 530

Q's

6.2 #17

types? red, white, blue

cardinality? unlimited

$$\underbrace{(x^0+x^1+x^2+\dots)}_{\text{red}}\underbrace{(x^0+x^1+x^2+\dots)}_{\text{white}}\underbrace{(x^0+x^1+x^2+\dots)}_{\text{blue}}$$

a) at least 2 $(x^2+x^3+\dots)^3$

b) $(x^0+x^1+x^2)\boxed{(x^0+x^1+x^2+\dots)^2}$ find

$\boxed{g(x)} = a_0x^0+a_1x^1+a_2x^2+\dots+\boxed{a_{10}}x^{10}+\dots$

(2) $(x^0+x^1+x^2)\left(\frac{1}{(1-x)^2}\right)$

(5) $(x^0+x^1+x^2)\left[1+\boxed{\binom{1+2-1}{1}x^1}+\binom{2+2-1}{2}x^2+\dots\right]$

$(x^0, x^1, x^2)\{x^0, x^1, x^2, \dots, x^8, x^9, x^{10}\}$

Ch 7 Recurrence Relations

Induction: (based upon a tautology)

Propositional function: P(element) → true / → false

Well ordered: case #1, case #2, case #3, ...

IF

$\underbrace{P(\text{case } \#1)}_{\text{Basis Step}} \wedge \underbrace{\forall_k \left(\left(P(1^{st}) \wedge P(2^{nd}) \wedge \cdots \wedge P(k-1^{st}) \right) \Rightarrow P(k) \right)}_{\text{Inductive Step}}$, then for all n, $P(n^{th} \text{ case})$

Call Basis Step to be $\boxed{\text{Initial Conditions}}$

Call Inductive Step to be a $\boxed{\text{recurrence relation}}$

So recurrence relations are:

given $\underbrace{a_0, a_1, a_2, \ldots, a_{k-1}}_{\text{old "things"}} \xrightarrow{\text{function}} \underbrace{a_k}_{\text{"new" thing}}$

$$a_n = f(a_0, a_1, \ldots, a_{n-1})$$

ex

Initial Value

$a_1 = 1$

$a_n = 2a_{n-1}$

open forms for the seq

recurrence relation

$1, 2, 4, 8, 16, 32, \ldots$

$n=1 \quad n=2 \quad n=3 \quad n=4 \quad n=5 \quad n=6$

closed form $= f(n)$

$a_n = 2^{n-1}$ closed

Why recurrence relations?

#1 Modeling

we usually make new from old!

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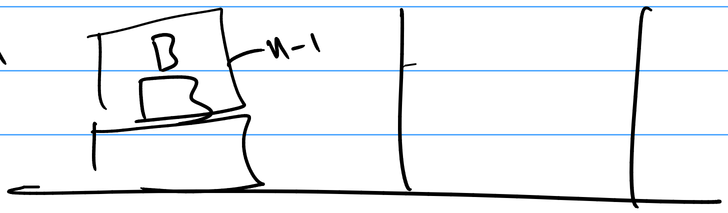
ex $\rightarrow a_0 = 0 \quad a_1 = 1$

$\rightarrow a_n = a_{n-1} + a_{n-2}$

$\rightarrow \boxed{0, 1, 1, 2, 3, 5, 8, \ldots}$

Tower of Hanoi.

n



$H_1 = 1$

$H_n = H_{n-1} + 1 + H_{n-1}$

$H_n = 2H_{n-1} + 1$

ex $\boxed{\text{red}}$, $\boxed{\text{blue}}$, $\boxed{\text{green}}$ tiles

tile n - spots

a_n

$\boxed{b}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ } \cdots \boxed{\ }$

$1 \quad \underbrace{2 \quad 3 \quad 4 \quad 5 \quad \cdots \quad n}_{a_{n-1}}$

$$a_n = 1 \cdot a_{n-1} + 1 \cdot a_{n-1} + 1 \cdot a_{n-1} = 3 a_{n-1}$$

1st tile

$$\boxed{\begin{aligned} a_1 &= 3 \\ a_n &= 3 a_{n-1} \end{aligned}}$$

$3, 3^2 = 9, 3^3 = 27, \ldots, 3^n, \ldots$

$a_1 \quad a_2 \quad a_3 \quad a_n$

Catalan Numbers

multiply $k+1$ things (using k multiplications)

$$((((n_0 \circ n_1) \circ n_2) \circ n_3) \circ \ldots) \circ n_k$$

exs 1 mult. $(n_0 \circ n_1)$

2 mult. $(n_0 \circ n_1) \circ n_2$ or $n_0 \circ (n_1 \circ n_2)$

3 mult. $((n_0 \circ n_1) \circ n_2) \circ n_3$, $(n_0 \circ n_1) \circ (n_2 \circ n_3)$, $n_0 \circ ((n_1 \circ n_2) \circ n_3)$

$(n_0 \circ (n_1 \circ n_2)) \circ n_3$, $n_0 \circ (n_1 \circ (n_2 \circ n_3))$

Initial Conditions $a_0 = 1$ $\quad (n_0)$

$a_1 = 1$ $\quad (n_0 \circ n_1)$

a_k ?

$(n_0 \circ n_1) \circ (n_2 \circ \cdots \circ n_k)$

$$a_k = a_0 a_{k-1} + a_1 a_{k-2} + \cdots + a_{k-1} a_0$$

$1 \cdot 1 = 1$ $\qquad 1, 1, 2, 5$

$1 \cdot 1 + 1 \cdot 1 = 2$

$1 \cdot 2 + 1 \cdot 1 + 2 \cdot 1 = 5$

$1 \cdot 5 + 1 \cdot 2 + 2 \cdot 1 + 5 \cdot 1 = 14$

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functions of two variables $\quad a_{n,k} = a(n,k)$

ex Select k objects of n distinct objects

n objects $= \boxed{O_1}, O_2, O_3, \ldots, O_n$

$$\boxed{a_{n,k} = 1 \cdot a_{n-1,k-1} + a_{n-1,k}}$$

Initial Conditions $\quad a_{n,0} = 1 \; ; \; a_{n,n} = 1$

and $\quad a_{n,k} = 0 \quad k > n$

$$a_{n,k} = \binom{n}{k}$$

$$\begin{array}{c|cccccc}
n \downarrow \; k \rightarrow & 0 & 1 & 2 & 3 & 4 & \cdots \\
\hline
0 & a_{0,0} & a_{0,1} & & & & \\
1 & a_{1,0} & a_{1,1} & a_{1,2} & \cdots & & \\
2 & & & & & & \\
3 & & & & & & \\
4 & & & & & & \\
5 & & & & & & \\
\vdots & & & & & &
\end{array}$$