Math 530

Grading?

5 pts (a) roll 10 times

$6 \cdot 6 \cdots 6 = 6^{10}$

$\cancel{\underbrace{6 \cdot 6 \cdots 6}_{10 \text{ times}} = 6^{10}}$

$10!$ 3/5 ? 2/5 ?

5 pts (b) 6-1's, 2-2's (string of rolls)

$$\frac{10!}{6!\,2!\,1!\,1!}$$

$10!$ ← arrangement of roll outcomes

$\boxed{Q\text{'s}}$

ex choose 4 of 3 types.

$$\binom{r+n-1}{r} = \binom{3+3-1}{4} = ?$$

?

8/40

$\boxed{Ch\,7}$ recurrence relations

ex (polys) $= a_0 x^0 + a_1 x^1 + a_2 x^2 + \cdots + a_r x^r + \cdots$

$$\boxed{a_r} = \boxed{}$$

\# of ways to select r objects

Making a_n

open form / recurrence relation

base elements
$$\begin{cases} a_0 = \\ a_1 = \\ \vdots \\ a_k = \end{cases}$$

k+1 on

$$a_{n+1} = f(a_0, \dots, a_n)$$

make new value

old values

Fibbonaci

$$\begin{cases} a_0 = 0 \quad a_1 = 1 \\ a_{n+1} = a_{n-1} + a_n \end{cases} \quad n = 1, 2, \dots$$

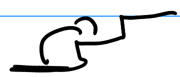
or

$$\begin{cases} a_0 = 0 \quad a_1 = 1 \\ a_n = a_{n-1} + a_{n-2} \end{cases}$$

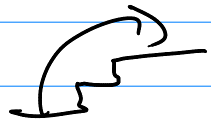
---

example: walk up the stairs to my office.

lazy → take 1 step



not lazy → take 2 steps



a_n = # of ways to go up n-stairs

1 step

$a_1 = 1$

0 step

$a_0 = 1$

$a_0 = 1, a_1 = 1, a_2 = ?, a_3 = ? \ldots$

$$a_n = \underbrace{1 \cdot a_{n-1}}_{\text{1-step at once}} + \underbrace{1 \cdot a_{n-2}}_{\text{2-steps at once}}$$

or

degree 2

open form

$$a_n = a_{n-1} + a_{n-2}$$

$$a_0 = 1 \quad a_1 = 1$$

$1, 1, 2, 3, 5, 8, \ldots$

closed forms

functions that form a_n

$$a_n = f(n)$$

ex

$$a_0 = 0 \quad a_1 = 1$$

$$a_n = a_{n-1} + a_{n-2}$$

seq.

$1, 1, 2, 3, \ldots$

also has

$$a_n = f(n) = \frac{1}{\sqrt{5}} \left(\boxed{\frac{1+\sqrt{5}}{2}} \right)^n - \frac{1}{\sqrt{5}} \left(\boxed{\frac{1-\sqrt{5}}{2}} \right)^n$$

$$a_1 = \frac{1}{2\sqrt{5}} \left(1 + \sqrt{5} \right) - \frac{1}{2\sqrt{5}} \left(1 - \sqrt{5} \right) = 1$$

Seq: $a_0, a_1, a_2, a_3, \ldots$

open form

$a_n = f(a_0, \ldots, a_{n-1})$

closed form

$a_n = f(n)$

Solve means find $a_n = f(n)$

How to solve?

#1 technique: guess and check

#2 technique: forward / backward iteration

ex

$a_0 = 1$
$a_n = 2a_{n-1}$

$a_0 = 1 = 2^0$
$a_1 = 2a_0 = 2(1) = 2^1$
$a_2 = 2a_1 = 2(2(1)) = 2 \cdot 2 \cdot 1 = 2^2$
$a_3 = 2a_2 = 2(2 \cdot 2 \cdot 1) = 2 \cdot 2 \cdot 2 \cdot 1 = 2^3$
$a_4 = 2a_3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 1 = 2^4$

$a_n = 2^n$

check?

relation

$a_n = 2a_{n-1}$

guess $a_n = 2^n$

check

$2^n \stackrel{?}{=} 2 \cdot 2^{n-1}$

$2^n \stackrel{?}{=} 2^n$

true

---

ex) Diff Eqn's $\boxed{y'' + y = 0}$

guess $y(x) = a\sin(x) + b\cos(x)$

check: $y''(x) = -a\sin(x) - b\cos(x)$

so $y'' + y' \stackrel{?}{=} 0$

7.2 | 7.3

7.3) $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$

recurrence relation: - linear
- homogeneous — term $(\text{coef})(a_i)$
- degree k
- constant coeff.

ex) $a_n = a_{n-1} + 3a_{n-2} - a_{n-3}$

$a_n = 2a_{n-5} + 10a_{n-11}$

looks like $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$

has exponential closed form

<u>guess:</u> $a_n = r^n$ $\quad$ <u>$r =$ constant.</u>

<u>if so</u> $\Rightarrow$ $r^n = c_1 r^{n-1} + c_2 r^{n-2} + \dots + c_k r^{n-k}$

$$1\cancel{r^n} = c_1 \frac{\cancel{r^n}}{r} + c_2 \frac{\cancel{r^n}}{r^2} + \dots + c_k \frac{\cancel{r^n}}{r^k}$$

$$r^k = c_1 r^{k-1} + c_2 r^{k-2} + \dots + c_k$$

$$\boxed{r^k - c_1 r^{k-1} - c_2 r^{k-2} - \dots - c_k} = 0$$

<u>characteristic polynomial</u> of degree k

has k solutions $r_1, r_2, \dots, r_k$

ex $\quad a_n = 1 \cdot a_{n-1} + 1 \cdot a_{n-2}$ $\quad$ <u>try:</u> $a_n = r^n$

characteristic polynomial

$$r^2 - 1r - 1 = 0$$

$$r = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

<u>have two r's</u> $\quad r_1 = \frac{1+\sqrt{5}}{2} \quad r_2 = \frac{1-\sqrt{5}}{2}$

So we found $a_n = a_{n-1} + a_{n-2}$

$a_n = \left(\frac{1+\sqrt{5}}{2}\right)^n$ or $\left(\frac{1-\sqrt{5}}{2}\right)^n$

remember multiplicity of a root

ex roots are $a_1 = 2$ $a_2 = 2$ $a_3 = 3$

2 has multiplicity 2

$a_n = (\alpha_0 + \alpha_1 n) 2^n + (\beta_0) 3^n$

poly of n's

poly of n's

ex

$\alpha_0 + \alpha_1 n + \alpha_2 n^2 + \alpha_3 n^3 + \dots$

etc

So $\left(\frac{1+\sqrt{5}}{2}\right), \left(\frac{1-\sqrt{5}}{2}\right)$

$a_n = (c)\left(\frac{1+\sqrt{5}}{2}\right)^n + (d)\left(\frac{1-\sqrt{5}}{2}\right)^n$

to find c, d use $\boxed{\text{bases}}$!

$n=0 \rightarrow$ $\underset{a_0}{0} = c + d$

$n=1 \rightarrow$ $\underset{a_1}{1} = c\left(\frac{1+\sqrt{5}}{2}\right) + d\left(\frac{1-\sqrt{5}}{2}\right)$

so solve! for c,d