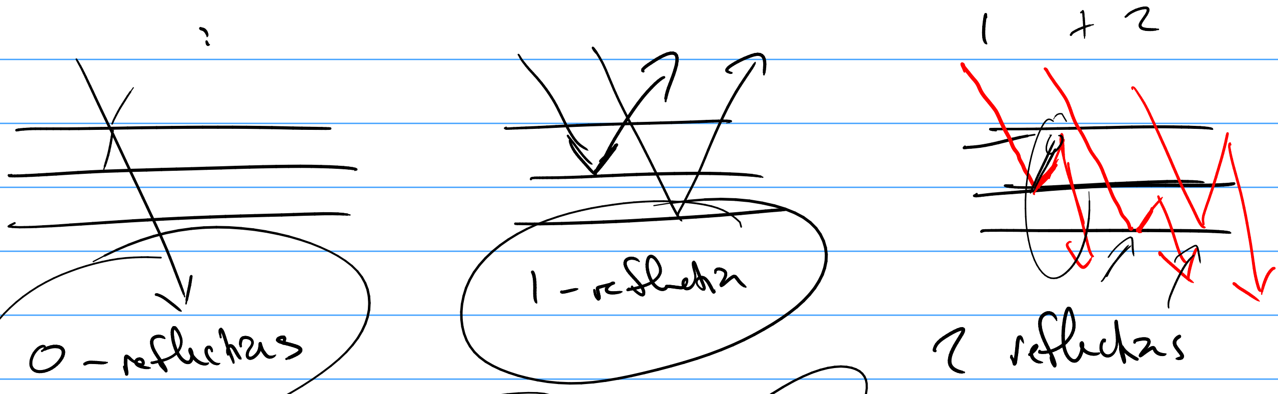
Math 530

Q's 7.1 (11) 7.2 (1d)

7.1 (11) $a_n = f(a_{n-1}, a_{n-2}, a_{n-3}, \ldots)$

$a_n \equiv$ what does this count?

a_{n-1}, a_{n-2}, ⋮ How do these relate?



bottom $a_n = a_{n-2} + (1) a_{n-1}$
cut bottom cut top

7.1 example 7 no $\boxed{012}$ allowed $t_i \in \{0,1,2\}$

a_n – string of length n | $t_1\ t_2\ t_3\ t_4\ \ldots\ t_n$

a_{n-1} ~ ways to have string of $n-1$ w/o 012 start w/ 2 and the rest of a_{n-1}

$a_n = \{1\} \cdot a_{n-1}$

or +

start w/ 1 → $\{1\} \cdot a_{n-1}$

or +

start w/ 0 → $\{1\} \cdot \boxed{a_{n-1}}$ no 012 in this if it starts with 12 then these are bad!

0 | $\boxed{12 \{ a_{n-3} \}}$

$a_n = a_{n-1} + a_{n-1} + a_{n-1} - a_{n-3}$

$$\boxed{a_n = 3 a_{n-1} - a_{n-3}}$$

start w/ 0 or 1 or 2

but overcount of 0 | 12 | a_{n-3}

1 way

$a_0 = 1$ $a_1 = 3$ $a_2 = 9$ $a_3 =$

$\{\lambda\}$

0, 1, 2

1 1 1

00	10	21
01	11	21
02	12	21

7.2 (1d)

table to know $\quad a_n = c\,a_{n/k} + f(n)$

① $a_n = a_{n/k} + d \xrightarrow{\text{solve}} a_n = d\lceil \log_k n \rceil + A$

② $a_n = k\,a_{n/k} + d \longrightarrow a_n = An - \frac{d}{k-1}$

③ $a_n = k\,a_{n/k} + dn \longrightarrow a_n = dn(\lceil \log_k n \rceil + A)$

④ $a_n = c\,a_{n/k} + dn \longrightarrow a_n = A\,n^{\log_k c} + \left(\frac{kd}{k-c}\right)n$

$c \neq k$

1d) $a_n = 3a_{n/3} + 4 \xrightarrow[\#2]{\text{table}} a_n = An - \frac{4}{3-1}$

$\boxed{a_n = An - 2}$

7.5 $\quad$ review table 6.1 of Section 6.2

→ finding coef. for generating functions

(polynomial, rational expressions) $= a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots$

generating function $\quad g(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots$

1^{st} differential equations

in the expression we have derivatives

expression = expression

$$y' + 2y'' = 3x + xy$$

2^{nd} Implicit functions

$$x + y^2 + yx = 3$$

but $\underbrace{y = f(x)}_{\text{explicit}}$

$z = f(x,y)$

ex
$$e^{xyz} = 2$$
$$\ln(e^{xyz}) = \ln(2)$$
$$xyz = \ln(2)$$
$$z = \frac{\ln(2)}{xy}$$

ex
$$x^2 + y^2 = 4$$
$$y^2 = 4 - x^2$$
$$y = \sqrt{4-x^2} \qquad y = -\sqrt{4-x^2}$$

3^{rd} functional equation

Solve!

$x^2 + y^2 = 4$ let $y = g(x)$

$x^2 + (g(x))^2 = 4$

$g(x) = \pm\sqrt{4-x^2}$

$g_1(x) = \sqrt{4-x^2}$ $g_2(x) = -\sqrt{4-x^2}$

$g(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n + \cdots$

tie with

rec. relation $a_n = f(a_{n-1}, a_{n-2}, \ldots, a_{n-k})$

initial values

$a_0 =$
$a_1 =$
$\vdots$
$a_k =$

"need" k initial values

degree k

use the same a_n's

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example 1 $g(x) = a_0 + a_1 x + a_2 x^2 + \cdots$

coef are recursively defined by

$$\begin{cases} a_0 = 1 \\ a_n = a_{n-1} + n \ ; \quad n = 1, 2, 3, \ldots \end{cases}$$

use rec. relation

$a_0 = 1$

$a_1 = a_0 + 1$

$a_2 = a_1 + 2$

$a_3 = a_2 + 3$

$a_4 = a_3 + 4$

$+1 \quad +2 \quad +3 \quad +4$

$g(x) = 1 + 2x + 4x^2 + 7x^3 + 11x^4 + 16x^5 + \dots$

$1+2+3+4 = \frac{4(5)}{2} = 10$

$\binom{5}{2}$

Solve by using generating functions?

rec rel. deg k

Initial values $a_0, a_1, \dots, a_k$ $\qquad a_n = f(a_{n-1}, a_{n-2}, \dots a_{n-k})$

$g(x) = \underbrace{a_0 + a_1 x + a_2 x^2 + \dots + a_k x^k}_{\text{known by initial values}} + \dots + a_n x^n + \dots$

$g(x) - [a_0 + a_1 x + \dots + a_k x^k] = a_{k+1} x^{k+1} + a_{k+2} x^{k+2} + \dots + a_n x^n + \dots$

Notice: $g(x) = \sum_{i=0}^{\infty} a_i x^i$ $\qquad \sum_{i=k+1}^{\infty} \boxed{a_i} x^i$

$a_i = f(a_{i-1}, a_{i-2}, \dots)$

$g(x) = a_0 + a_1 x + a_2 x^2 + \dots$

$g(x) - a_0 = a_1 x + a_2 x^2 + \dots = \sum_{n=1}^{\infty} \boxed{a_n} x^n$ $\qquad a_{n-1} + n$

$g(x) - a_0 = \sum_{n=1}^{\infty} (a_{n-1} + n) x^n$

$g(x) - a_0 = \sum_{n=1}^{\infty} a_{n-1} x^n + \sum_{n=1}^{\infty} n x^n$

$\vdots$

$$g(x) - a_0 = x\,g(x) + \sum_{n=1}^{\infty} \binom{n}{1} x^n$$

$$\boxed{g(x) - a_0 = x\,g(x) + \frac{x}{(1-x)^2}}$$

by section 6.2

$$g(x) = (\qquad)$$