

# Math 530

$$|A_1 A_2 A_3 \dots A_n| = |U| - S_1 + S_2 - S_3 + \dots + (-1)^n S_n$$



## Derangements

String  $p_1 p_2 p_3 \dots p_n$

all strings such that each symbol in the old string is not in its old place.

$$|\overline{A_1} \overline{A_2} \dots \overline{A_n}|$$

$\overline{A_1} \equiv$  symbol 1 is not in old place  
 $A_1 \equiv$  symbol 1 is in its old place

$$D_n = |\overline{A_1} \overline{A_2} \dots \overline{A_n}|$$

$$= |U| \rightarrow n!$$

$$- S_1 \rightarrow S_1 = |A_1| + |A_2| + \dots + |A_n| = (n-1)! + \dots + (n-1)! = n(n-1)! = n!$$

$$+ S_2 \rightarrow S_2 = |A_1 A_2| + |A_1 A_3| + \dots + |A_{n-1} A_n| = (n-2)! + (n-2)! + \dots + (n-2)! = \binom{n}{2} (n-2)!$$

$$- S_3$$

$$S_0, S_k = \binom{n}{k} (n-k)!$$

$$D_n = \binom{n}{0} (n-0)! - \binom{n}{1} (n-1)! + \binom{n}{2} (n-2)! - \dots + (-1)^n \binom{n}{n} 0!$$

$$D_n = \sum_{k=0}^n (-1)^k \binom{n}{k} (n-k)! = \sum_{k=0}^n (-1)^k \frac{n!}{k!}$$

$$D_n = n! \sum_{k=0}^n \frac{(-1)^k}{k!} \approx n! e^{-1} = \frac{n!}{e}$$

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k = 1 + x + \frac{1}{2!} x^2 + \frac{1}{3!} x^3 + \dots$$

7.5 example 1

$$(4) = x \sum_{n=0}^{\infty} a_n x^n + \sum_{n=0}^{\infty} \binom{n}{1} x^n$$

$$\sum_{n=0}^{\infty} \binom{n}{1} x^n = \cancel{\binom{0}{1} x^0} + \binom{1}{1} x^1 + \binom{2}{1} x^2 + \binom{3}{1} x^3 + \dots$$

$$\frac{x}{(1-x)^2} = x + \binom{2}{1} x^2 + \binom{3}{2} x^3 + \binom{4}{3} x^4 + \dots$$

$$\frac{1}{(1-x)^n} = 1 + \binom{1+n-1}{1} x + \binom{2+n-1}{2} x^2 + \binom{3+n-1}{3} x^3 + \dots$$

$$\binom{k}{1} = \binom{k}{k-1}$$

$$\binom{1+n-1}{1} = \binom{1+n-1}{n-1}$$

$$\binom{2+n-1}{2} = \binom{2+n-1}{n-1}$$

Exam 12 probs @ (0pts / 110pts = 100%)

7.1 Recurrence Relations 3 problems  
correct

① given an answer  $\rightarrow$  you explain it.

② } word probs like #5, #9 & HW  
③ } also use rec. relata to find soln of the seq.

$$a_n = 2a_{n-1} + a_{n-3}$$

$$a_0 = 1$$

$$a_1 = 1$$

$$a_2 = 2$$

1, 1, 2, 5, 11, 24, ...

7.2 Divide and Conq. 1 prob

① given  $a_n = 2a_{n/k}$   
expression

use table to get ans. and check.

Soln  $a_n = 5^n$

(check)

$$\begin{aligned} a_n &= 5 a_{n-1} \\ 5^n &= 5 \cdot 5^{n-1} \\ 5^n &= 5^n \end{aligned}$$

trans

7.3

2 probs

Soln to linear homogeneous rec. relations of deg  $k$  with constant coeff.

$$\rightarrow a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$$

① given rec. relation  $\rightarrow$  solve

② word probs.  $\rightarrow$  make rec. relation & solve it

(ex)  $a_n = \dots$

$$r_1 = 3 \quad r_2 = 3 \quad r_3 = 3 \quad r_4 = -1 \quad r_5 = -1$$

$$a_n = (c_1 + c_2 n + c_3 n^2) (3)^n + (c_4 + c_5 n) (-1)^n$$

7.5

0 probs

extra credit

5 pts

also hand it a typed write up & 2.5 example 1

8.1

Venn Counting

3 probs

① explain & give answer.

② Pairing Questions like #3; #5

③ Venn Diagram like #11; #15

draw

