Math 530

Due Wed

1.1(1, 4, 15)
1.2(1, 3a, 5aceg)
1.3(1, 3, 7, 9)

Isomorphic Graphs

$G_1 = (V_1, E_1)$ $\quad$ $G_2 = (V_2, E_2)$

bijection from V_1 to V_2
<u>that</u> preserves edges

$\{a, b\} \in E_1$ then $\{f(a), f(b)\} \in E_2$

if $G_1 = (V_1, E_1)$

we can call $\overline{G_1} = (V_1, \overline{E_1})$

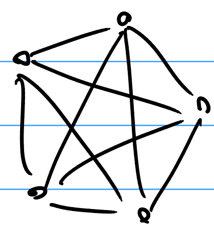
(simple) $\quad K_n$ is the complete graph (everyone is connected to every one)

$$A_{K_n} = \begin{bmatrix} 0 & & 1 \\ & \ddots & \\ 1 & & 0 \end{bmatrix}$$

1's everywhere else

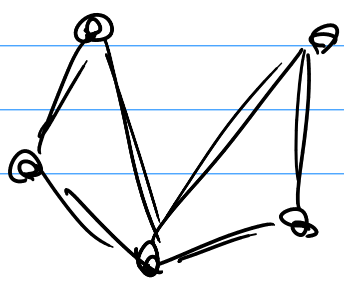
no loop

K_5



(ex) G is 5 vertices

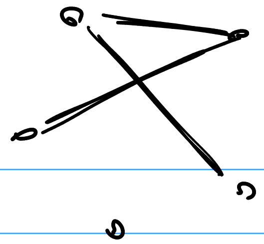
$G = (V, E)$



$\overline{G} = (V, \overline{E})$

$\overline{E}$: edges of K_n $- E$

$\overline{G} = (V, \overline{E})$

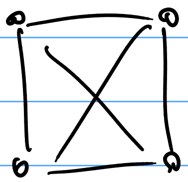


Th^n G_1 and G_2 are isomorphic

iff $\overline{G_1}$ and $\overline{G_2}$ are isomorphic

HW 1.2 #1 (all) non-isomorphic for $|V| = 4$

K_4



$|E| = 6$

Isomorphic for $|E| = 0$

Isomorphic for $|E| = 1$

$|E| = 2$

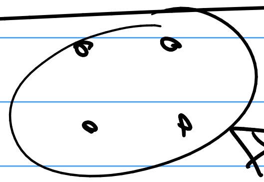
$|E| = 3$

$|E| = 4 \rightarrow |\overline{E}| = 2$

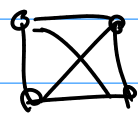
$|E| = 5$

$|E| = 6$

$|E| = 0$

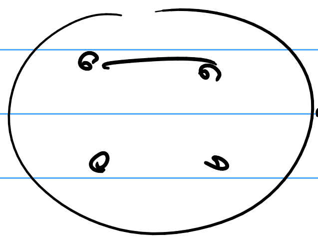


$|E| = 6$ is [complement

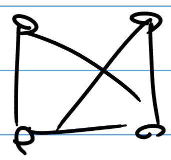


K_4

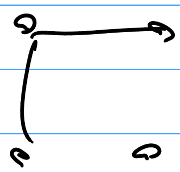
$|E| = 1$

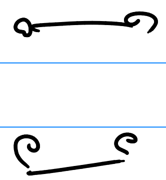


$\xrightarrow{\overline{G}}$



$|E| = 2$

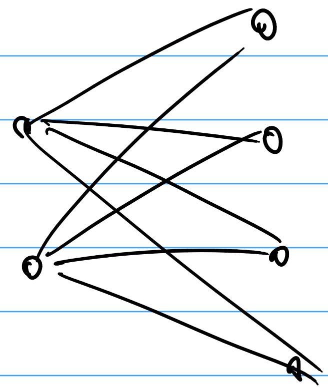




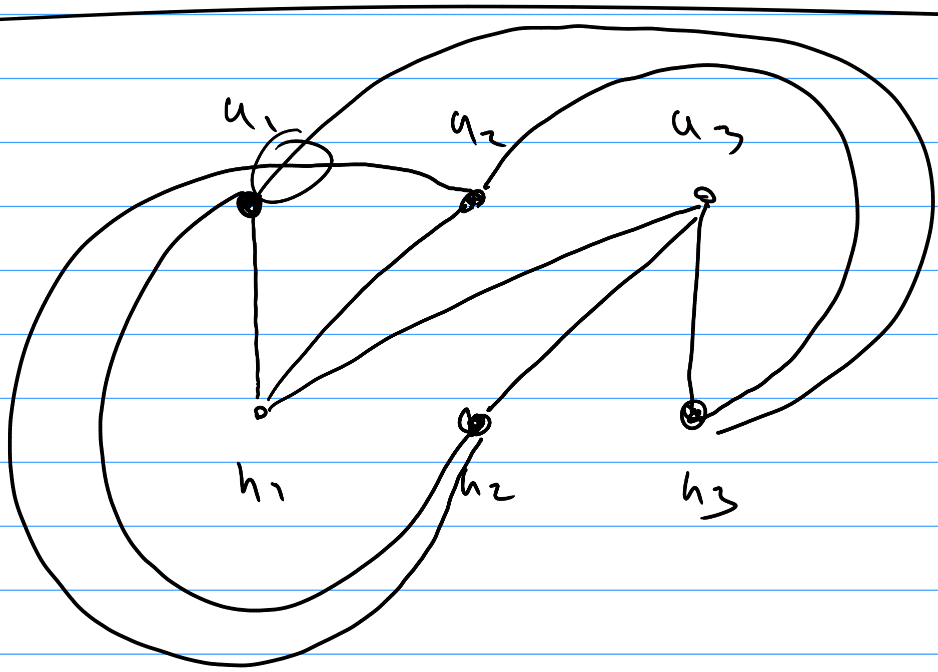
1.4 Planar Graphs

(ex) $K_{n,m}$ — n, m complete bipartite graph

(ex) $K_{2,4}$



$K_{3,3}$

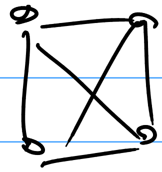


non planar

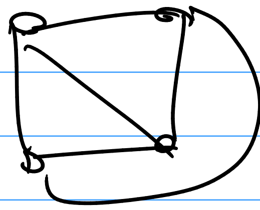
> If you can draw an isomorphic graph whose edges do not cross, then it is planar

ex K_4

planar



Isomorphic

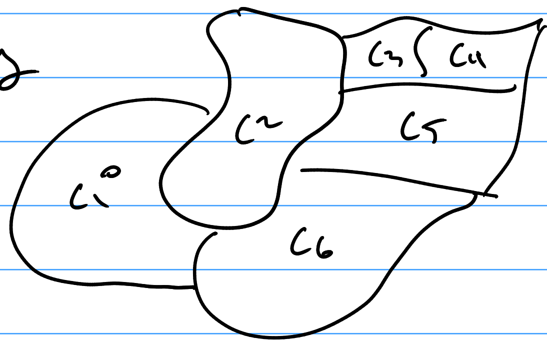


Note: K_5, $K_{3,3}$ are the two principle non-planar graphs.

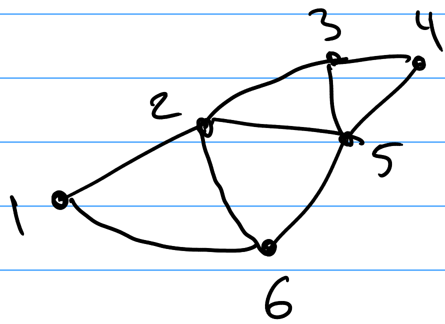
Thm A graph is non-planar iff it contains K_5 or $K_{3,3}$ as a subgraph.

Ex (for planar graphs)

Map making



dual graph



Note: Thm you need at most 4 colors to color a map.

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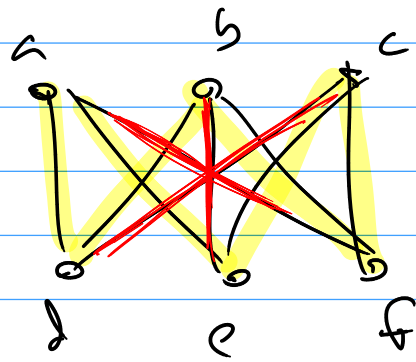
to test planar

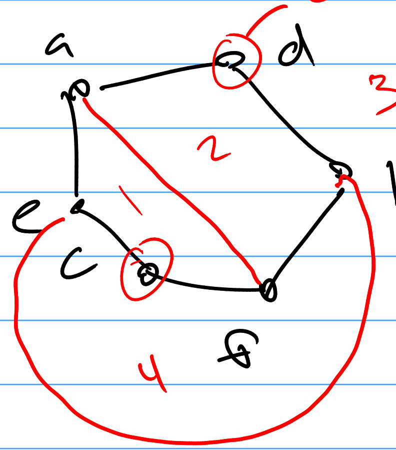
(assume a "typical" graph with lots of edges and lots of common degrees)

Look for Hamilton Circuit. To go through all vertices exactly once with a simple circuit.

$K_{3,3}$

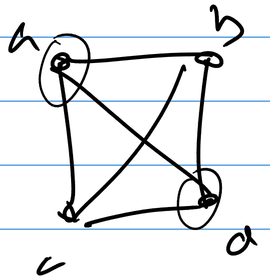


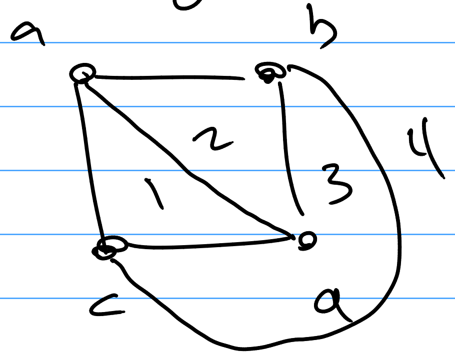
can't get to c



1. Make your circle (hamilton circuit)
2. start adding chords through common regions
3. - Planar if all edges are added and no crosses
   - non planar if you need to connect vertices w/o a common region

ex





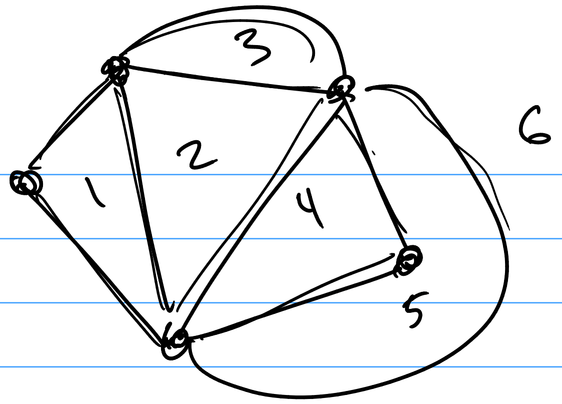
4

$|r| = 4$

$|V| = 4$] 8

$|E| = 6$

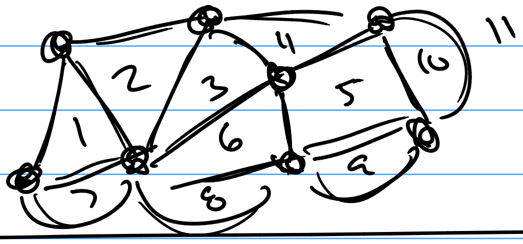
ex



$|r| = 6$
$|V| = 5$ $\Big] = 11$

$|E| = 9$

ex



$|r| = 11$
$|V| = 8$ $\Big] = 19$

$|E| = 17$

$\boxed{\text{th}^{m}}$ Euler's Formula (1752)

G is a connected planar graph

$$|r| + |V| = |E| + 2$$

$$|r| = |E| - |V| + 2$$

$$|E| = |r| + |V| - 2$$

Corollary if $|E| > 1$ then $|E| \leq 3|V| - 6$
