Math 530

Final Exam

4 problems each / exams 1 to 3

so -- 12 problems

- WSU Fields Schedule
  - Tues Dec 8th | 1pm - 3pm
- Our Schedule
  - Tues Dec 8th | exam is open 12pm - 4pm

2 hrs to do and upload the exam.

Study?

- exams 1 to 3.

Do all these exams and variations of problems

1. find section # for each problem
2. do similar problems from that section

MATH 530 ... EXAM 3

Key

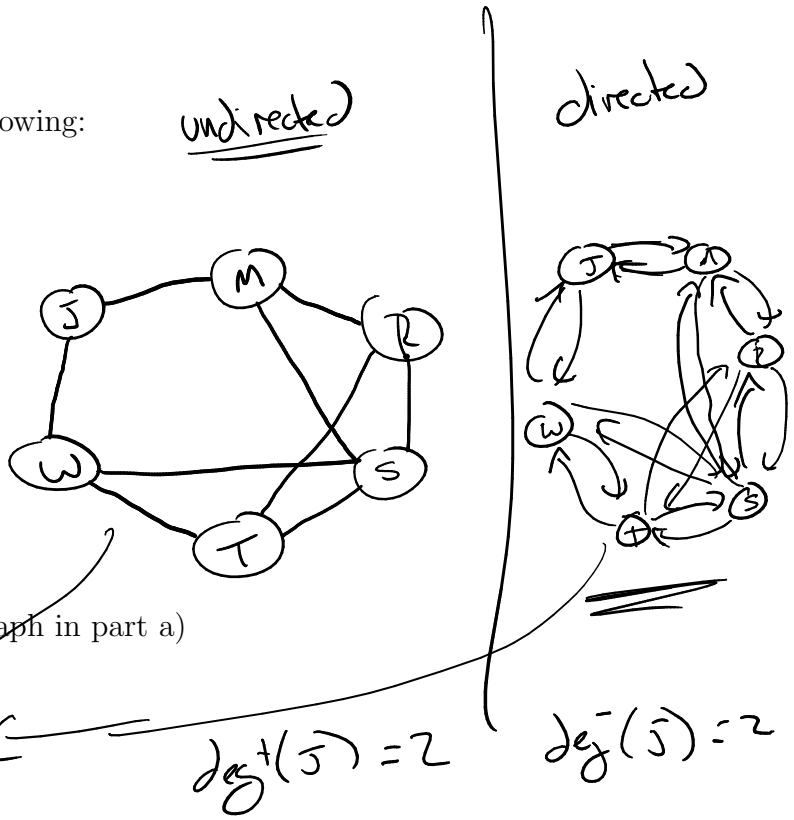
0) Exam Start Time:

0) MyWSUid and Name:

1) For problem 4 from section 1.1 answer the following:

a) Model the rumor-passing with a graph.

4. Suppose there are six people—John, Mary, Rose, Steve, Ted, and Wendy—who pass rumors among themselves. Each day John talks with Mary and Wendy; Mary talks with John, Rose, and Steve; Rose talks with Mary, Steve, and Ted; Steve talks with Mary, Rose, Ted, and Wendy; Ted talks with Rose, Steve, and Wendy; and Wendy talks with John, Steve, and Ted. Whatever people hear one day they pass on to others the next day.



b) Write the adjacency matrix, A_G for your graph in part a)

$$A_G = \begin{array}{c} \\ J \\ M \\ R \\ S \\ T \\ W \end{array} \begin{array}{c} \begin{array}{cccccc} J & M & R & S & T & W \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

$\deg^+(J) = 2$ $\deg^-(J) = 2$

c) If your graph is undirected give the degrees for each vertex. If your graph is directed give the in-degree and out-degree for each vertex.

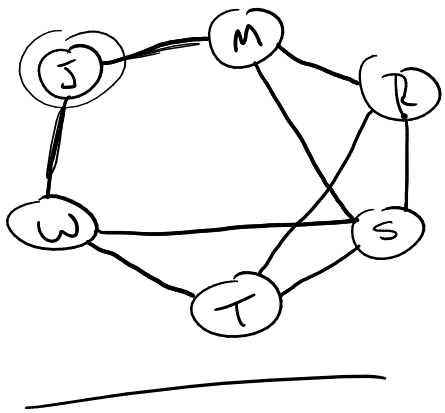
$\deg(J) = 2$
$\deg(M) = 3$
$\deg(R) = 3$
$\deg(S) = 4$
$\deg(T) = 3$
$\deg(W) = 3$

$\deg^+(J) = \deg^-(J) = 2$

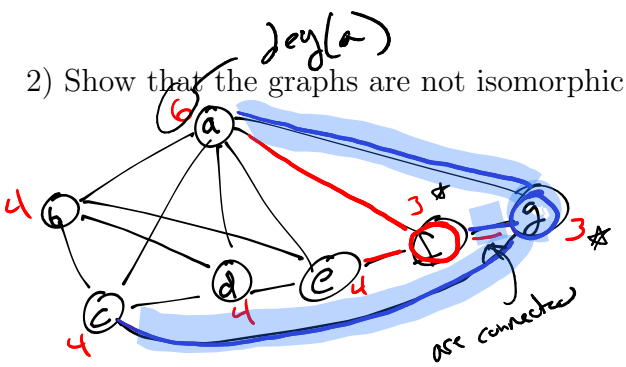
etc

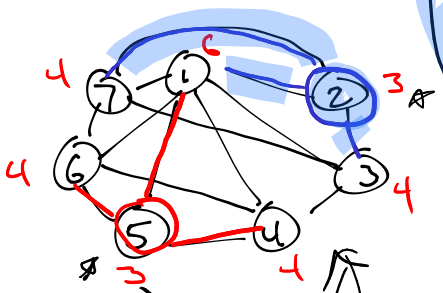
1

directed



2) Show that the graphs are not isomorphic.

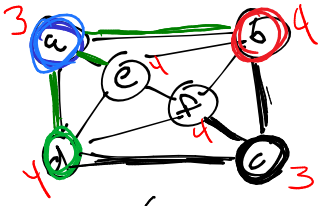


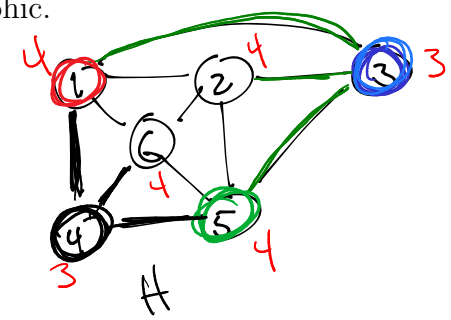


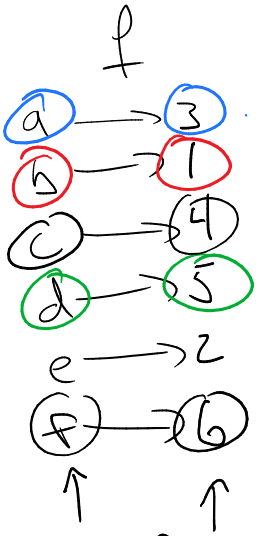
find most unig. features

deg(3) vertices are not connected

3) Show that the graphs are isomorphic.





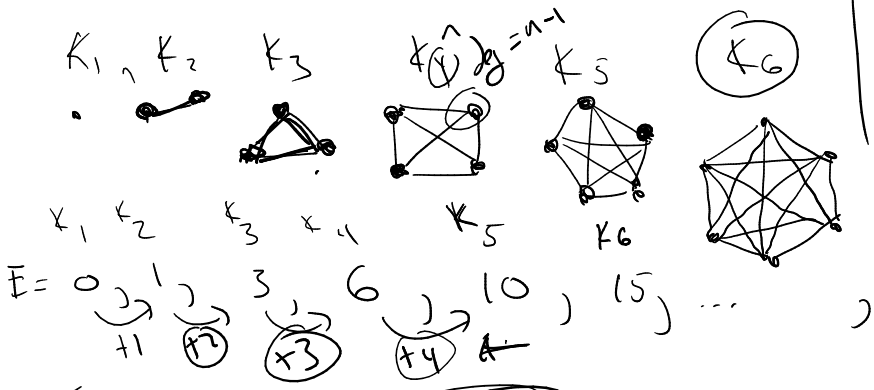


$A_G =$ $= A_H =$

Yes

So isomorphic and this is the isomorphism

4) If you are told that you have E edges in the K_n complete graph, how many vertices are in the graph?



K_n: $n(n-1)$ = sum of degrees

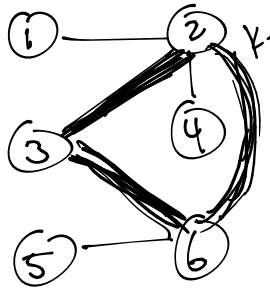
$\frac{n(n-1)}{2} = E \Leftrightarrow n(n-1) = 2E$

$n^2 - n - 2E = 0$

$n = \frac{1 \pm \sqrt{1+8E}}{2}$ ← why not $-$?

need pos n

5) Is the given graph bipartite? Explain your answer.



No. $6,2,3,6$ is a circuit of odd length.

by the bipartite iff all circuits are even length.

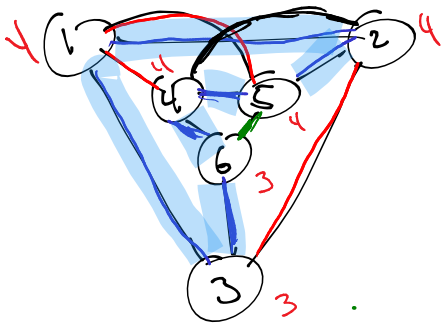
K_3 so at least 3 colors → not bipartite

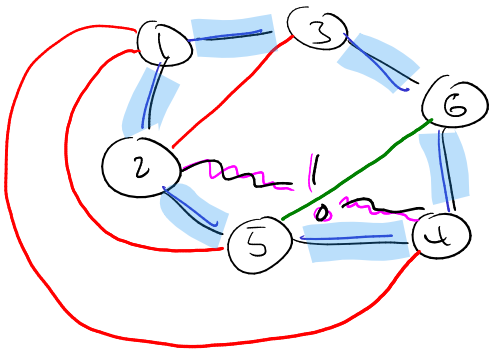
2 color

6) Is the graph planar? Find $K_{3,3}$ or K_5 configurations if it is nonplaner.

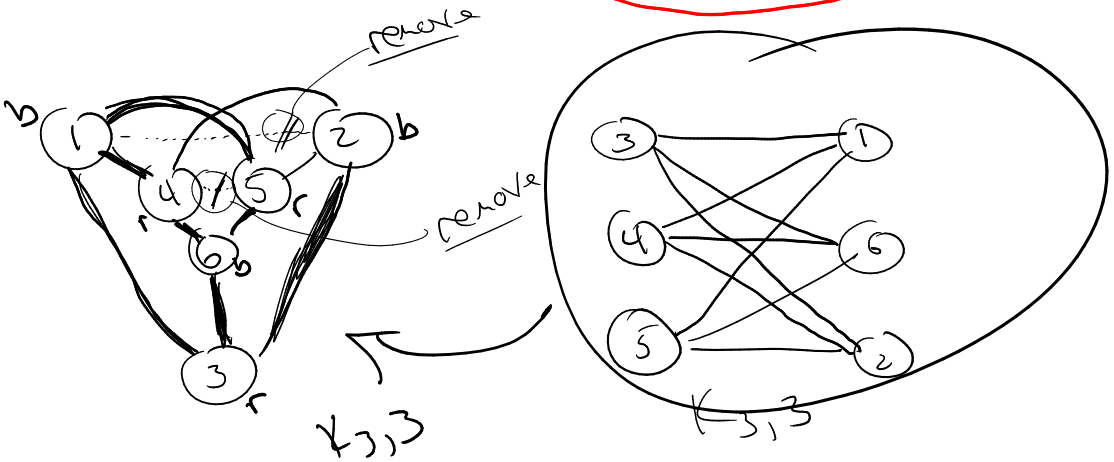
6 vert. deg 3

5 vert. deg(v)=4





nonplaner

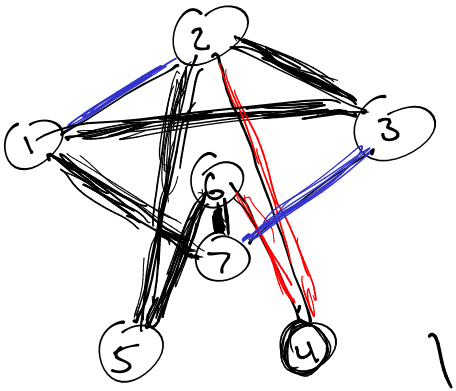


remove

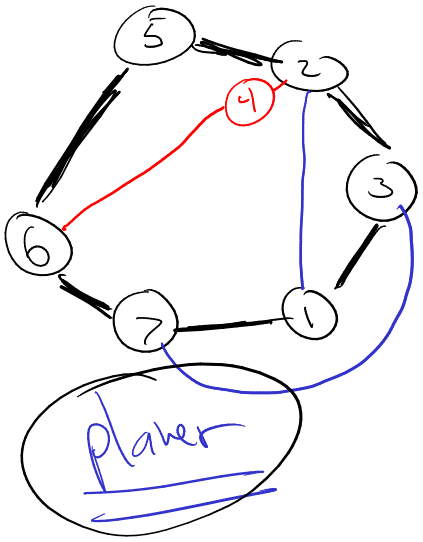
remove

$K_{3,3}$

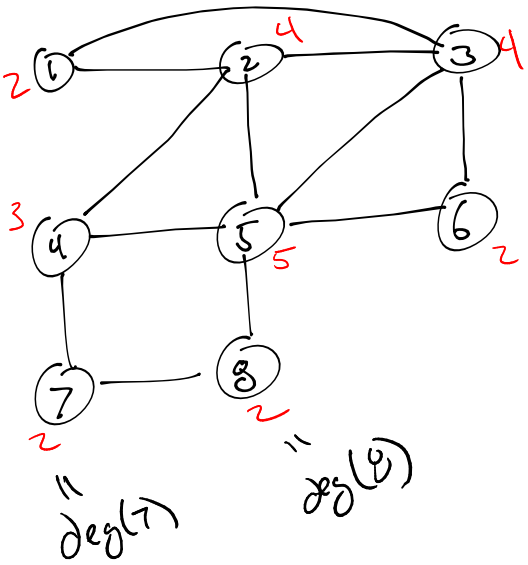
7) Is the graph planar? Find $K_{3,3}$ or K_5 configurations if it is nonplaner.



Many answers!



8) Does the given graph have an Euler circuit or path? Explain your answer.

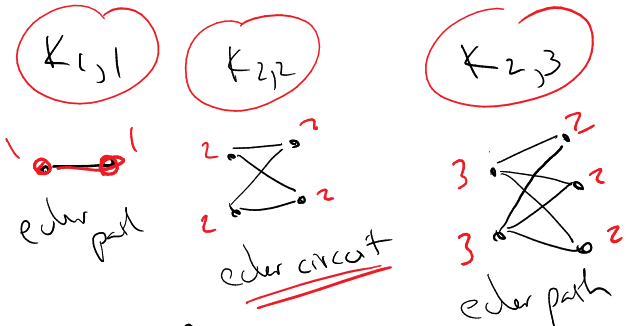


all even deg? → Euler Circuit

exactly 2 odd deg? → Euler path

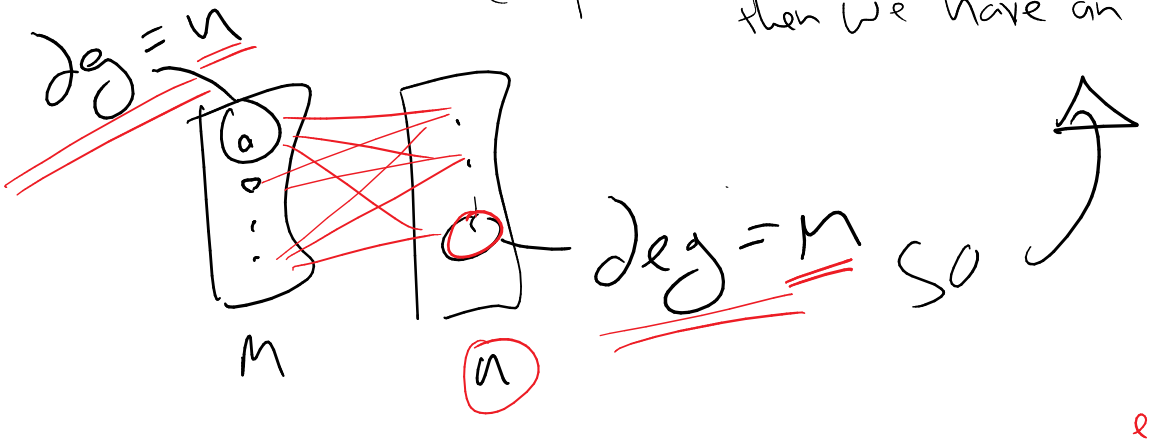
exactly 2 odd vertices so has an euler path.

9) When does the complete bipartite graph $K_{m,n}$ have an Euler circuit? When does it have an Euler path? Explain your answers.



(a) $K_{m,n}$ if both are even (all deg(v) are even) we have an euler circuit.

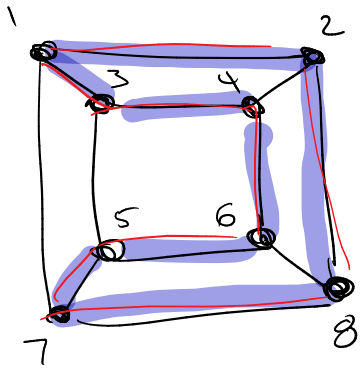
(b) If one side is 2 and the other odd, then we have an euler path.



exactly 2 that are odd deg.

exactly 2 odd number

10) Does Dirac's Theorem apply to the 3D cube Q_3? Does it have an Hamilton circuit?



$\deg(v) = 3 \neq 4/2 = 4 \quad \forall v \in V$

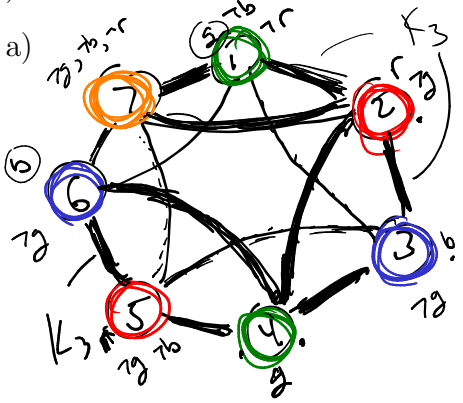
so NO Dirac's th^m does not apply

Dirac's: If $\deg(x) \geq \frac{|V|}{2}$ then Hamilton Circuit

but 1, 2, 8, 7, 5, 6, 4, 3, 1 is a hamilton circuit.

11) What are the chromatic numbers for the given graphs?

a)



Contains (subgraph) K_2, K_3, K_4, ...

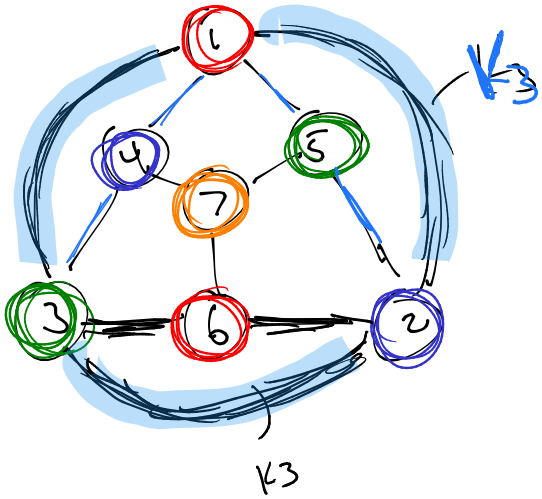
colors ≥ 2

≥ 3

≥ 4

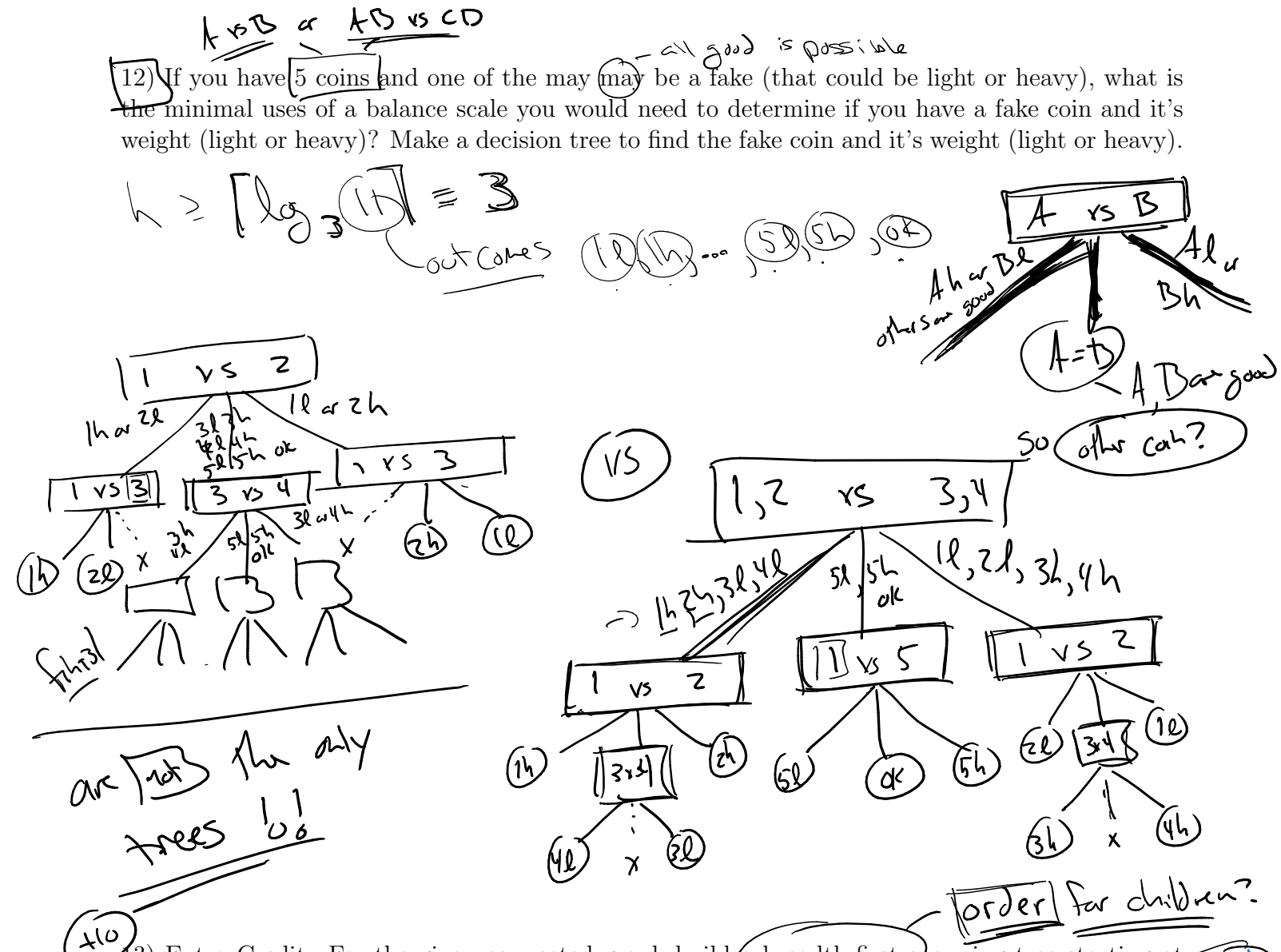
4

b)

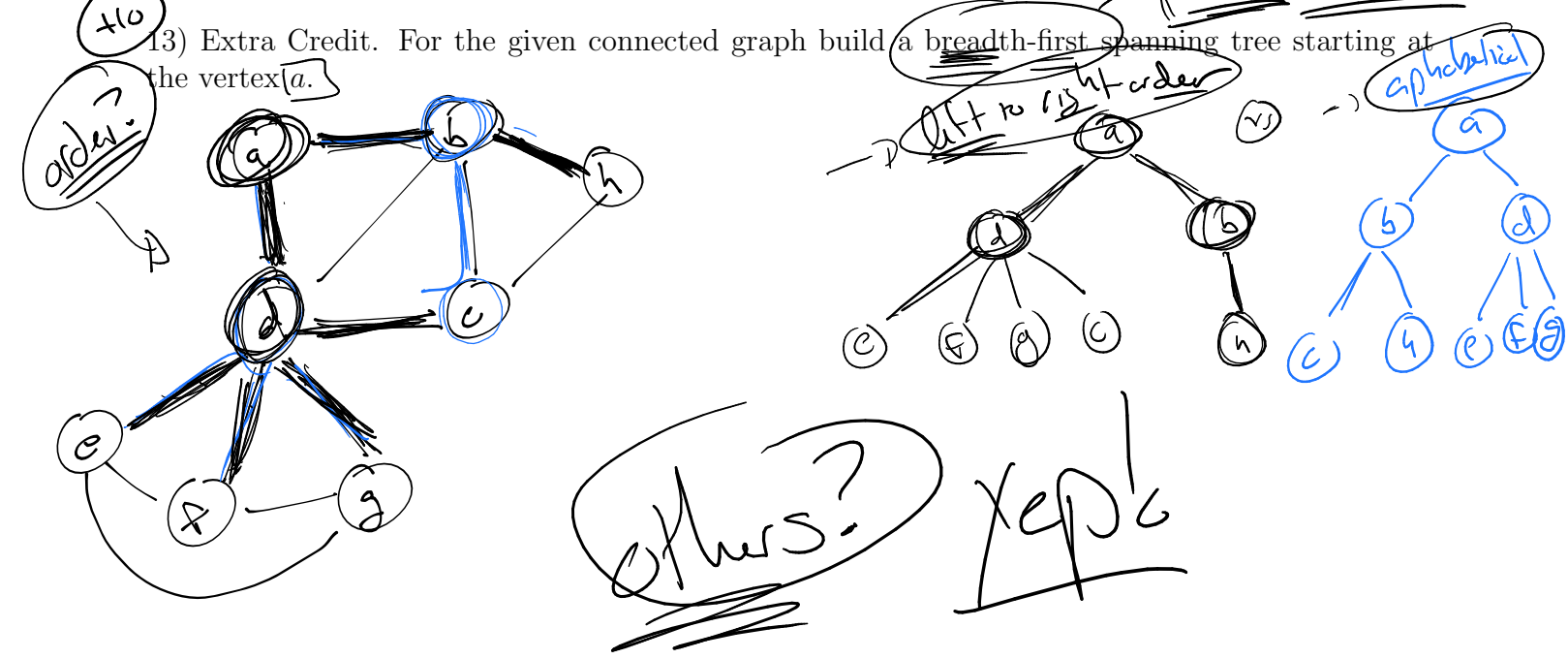


4

12) If you have 5 coins and one of the may may be a fake (that could be light or heavy), what is the minimal uses of a balance scale you would need to determine if you have a fake coin and it's weight (light or heavy)? Make a decision tree to find the fake coin and it's weight (light or heavy).



13) Extra Credit. For the given connected graph build a breadth-first spanning tree starting at the vertex a.



0) What is the time you ended working on the exam and started scanning it?