Math 322

Identity

$x + 0 = x$

$x \cdot 1 = x$

Domination

$x + 1 = 1$

$x \cdot 0 = 0$

x	y	$x+y$ (Maxterm)	$x \cdot y$ (Minterm)
1	1	1	1
1	0	1	0
0	1	1	0
0	0	0	0

uniq? $\boxed{\text{one is uniq}}$ b/c $x \cdot 0 = 0$ Domination law

uniq? $\boxed{\text{Zero is uniq.}}$ b/c $x + 1 = 1$ Domination law

literals: x or $\bar{x}$, y or $\bar{y}$

Minterms	x	y	$x \cdot y$	$\bar{x} \cdot y$	$x \cdot \bar{y}$	$\bar{x} \cdot \bar{y}$
xy	1	1	1	0 0 1	1 0 0	0 0 0
$x\bar{y}$	1	0	0	0 0 0	1 1 1	0 0 1
$\bar{x}y$	0	1	0	1 1 1	0 0 0	1 0 0
$\bar{x}\bar{y}$	0	0	0	1 0 0	0 0 1	1 1 1

Minterms focus on $\boxed{\text{one}}$!

q	s	
1	1	0
1	0	1 ← $q\bar{s}$
0	1	0
0	0	0

Maxterm → Focus on $\boxed{\text{Zero}}$!

	x	y	$x+y$	$x+\bar{y}$	$\bar{x}+y$	$\bar{x}+\bar{y}$
$\bar{x}+\bar{y}$	1	1	1	1 1 0	0 1 1	0
$\bar{x}+y$	1	0	1	1 1 1	0 0 0	1
$x+\bar{y}$	0	1	1	0 0 0	1 1 1	1
$x+y$	0	0	0	0 1 1	1 1 0	1

$\boxed{\text{ex}}$ Minterm expansion / Sum of products / Sum of minterms / disjunctive normal form

s	q	F	
1	1	$\boxed{1}$	$s \cdot q$
1	0	0	
0	1	0	
0	0	$\boxed{1}$	$\bar{s} \cdot \bar{q}$

$$\boxed{F(s,q) = s \cdot q + \bar{s} \cdot \bar{q}}$$

x	y	z	F	
1	1	1	0	
1	1	0	$\boxed{1}$	← $x \cdot y \cdot \bar{z}$
1	0	1	0	
1	0	0	$\boxed{1}$	← $x\bar{y}\bar{z}$
0	1	1	0	
0	1	0	0	
0	0	1	0	
0	0	0	$\boxed{1}$	← $\bar{x}\bar{y}\bar{z}$

$$\boxed{F(x,y,z) = xy\bar{z} + x\bar{y}\bar{z} + \bar{x}\bar{y}\bar{z}}$$

ex 3 product of sums | product of maxterms

Max term expansion | conjunctive normal form

x	y	z	F
1	1	1	$\boxed{0}$ ← $\bar{x}+\bar{y}+\bar{z}$
1	1	0	$\boxed{0}$ ← $\bar{x}+\bar{y}+z$
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	$\boxed{0}$ ← $x+y+\bar{z}$
0	0	0	1

$$\boxed{F(x,y,z) = (\bar{x}+\bar{y}+\bar{z})(\bar{x}+\bar{y}+z)(x+y+\bar{z})}$$

$F(x,y,z) = \boxed{xy} + \boxed{xz}$ Sum of products?

z? y?

$$= x \cdot y \cdot 1 + x \cdot 1 \cdot z$$
$$= xy(z+\bar{z}) + x(y+\bar{y})z$$
$$= xyz + xy\bar{z} + xyz + x\bar{y}z$$
$$= \boxed{xyz + xy\bar{z} + x\bar{y}z}$$

expansions

① use table

② use boolean algebra laws

Ex

$$F(x,y,z) = x + y$$
$$= (x+y) + (0)$$
$$= (x+y) + (z \cdot \bar{z})$$

Product of sums (maxterms)

$$(x \vee y) \vee (z \wedge \bar{z})$$
$$(x \vee y \vee z) \wedge (x \vee y \vee \bar{z})$$
$$= \boxed{(x+y+z) \cdot (x+y+\bar{z})}$$

Reminder:

$$x \cdot (y+z) = x \cdot y + x \cdot z$$
$$x + (y \cdot z) = (x+y) \cdot (x+z)$$

Tables:

x	y	F
1	1	
1	0	
0	1	
0	0	

$\rightarrow 2^4 = 16$ total uniq tables. for any of the functions

b/c min-term / max-term expansions handle all 16

only use $\cdot, +, \bar{\ }$

We call the set $\{\cdot, +, \bar{\ }\}$ to be functionally complete!

Can $\{\cdot, +, ^{-}\}$ be reduced? (and still be functionally complete)

Notice: $\overline{\overline{q}} = q$

$\overline{x+y} = \overline{x}\,\overline{y}$

$x+y = \boxed{\overline{\overline{x}\,\overline{y}}}$

given any $+ \rightarrow \{\cdot, ^{-}\}$ is functionally complete.

Note: $\overline{xy} = \overline{x} + \overline{y} \Rightarrow xy = \overline{\overline{x} + \overline{y}}$

given any $\cdot \rightarrow \{+, ^{-}\}$ is functionally complete

Def: $x \downarrow y = \overline{x+y}$

$\overline{x} = x \downarrow x$

$x+y = (x \downarrow y) \downarrow (x \downarrow y)$

$xy = (x \downarrow x) \downarrow (y \downarrow y)$

$\{\downarrow\}$ is functionally complete

Def: $x \mid y = \overline{xy}$

$\overline{x} = x \mid x$

$x+y = (x \mid x) \mid (y \mid y)$

$xy = (x \mid y) \mid (x \mid y)$

$\{\mid\}$ is functionally complete