Math 322

Relation: Subset of $A \times B$

$$R : A \to B$$

list: $R = \{ (a_1, b_1), \ldots \}$ $\quad a_1 R b_1$

Visualize: list (a_1, b_1) $\quad$ digraph $a_1 \longrightarrow b_1$

$$M_R = [\text{zero one matrix}]$$

Properties of Relations on set A

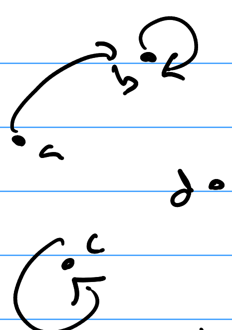
① **Reflexive:** R is called reflexive when $\forall e\,(eRe)$

② **Irreflexive** R is called irreflexive when $\forall e\,(e \not R e)$

Reflexive? digraph: all loop
Matrix: $\begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}$

Irreflexive? digraph: no loops
Matrix: $\begin{bmatrix} 0 & & \\ & \ddots & \\ & & 0 \end{bmatrix}$

ex's



$$M_R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

digraph $\qquad$ Matrix

not reflexive
not irreflexive

③ <u>Symmetric</u>   <u>Idea:</u> if $a \bullet \to \bullet\, b$

then must have $a \bullet \leftarrow \bullet\, b$

$$\forall a \forall b \, (\boxed{aRb} \to bRa)$$

<u>digraph?</u> $a \rightleftarrows b$

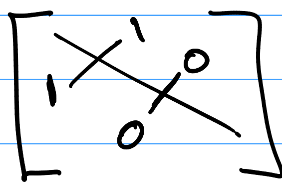
<u>a counter example:</u> $a \bullet \to \bullet\, b$

ex: empty relation

$\bullet\, a$ $\bullet\, b$ $\bullet\, c$ $\bullet\, d$

reflexive: no
irreflexive: yes
symmetric: yes

<u>Matrix:</u>



<u>symmetric across main diagonal</u>

$\Updownarrow$

$$M_R = M_R^T$$

ex $M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

ex R on all people

$R = \{ (a,b) \mid a, b$ have at least one common biological parent $\}$

<u>ref:</u> $\forall p \, (pRp)$ "everyone has at least one common biological parent with themselves"
<u>true.</u>

so <u>Reflexive</u>

<u>irreflexive:</u> $\forall p \, (p \not R p)$

<u>No</u> b/c Mark's biological parents (Mom/Dad) mean he has at least one. (counter ex)

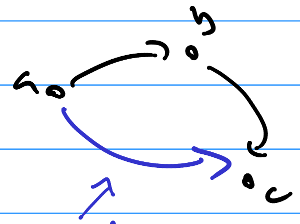
Symmetric $\forall a \forall b\,(aRb \to bRa)$

true: biological siblings are symmetric.

4 transitive $\forall a \forall b \forall c\,(aRb \wedge bRc \to aRc)$

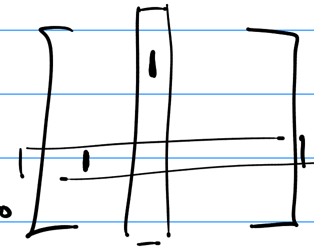
who goes to e_2

digraph:

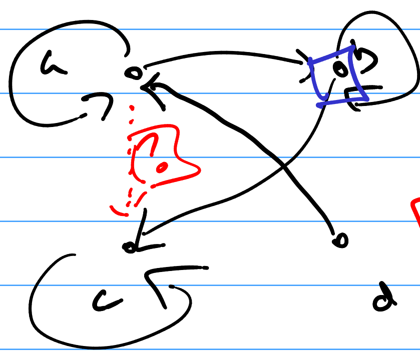


must have this relationship

Matrix:



e_1 goes to



No

see red counter example

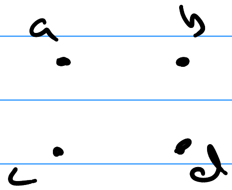
ex

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & \boxed{0} & 0 \\ b & 0 & 1 & 1 & 0 \\ c & 0 & 0 & 1 & 0 \\ d & 1 & 0 & 0 & 0 \end{array}$$

b goes to..

... go to b

Back to ..



$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

not ref.
is irref.
is sym
is trans.

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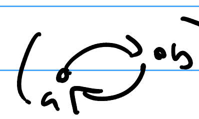
5 anti symmetric $\forall a \forall b\,(aRb \wedge bRa \to a=b)$

$\equiv \forall a \forall b\,(a \neq b \to a\not R b \vee b \not R a)$

$\equiv \forall a \forall b\,(a \neq b \to \neg(aRb \wedge bRa))$

$\neg($  $)$

6 asymmetric $\forall a \forall b\,(aRb \to b \not R a)$

$\equiv$ (irreflexive $\wedge$ anti symmetric)