Math 322

Due on day of exam
7.1 (1, 3, 7)
7.2 (1, 2, 4, 7, 9)
7.3 (1, 4, 8)

6.5 #3

#1 S on $\{0,1,2,3,4\}$

$$M_S = \begin{bmatrix} 0&1&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&1&0&0&0 \end{bmatrix}$$

$$M_{S^+} = \underbrace{M_S}_{\text{length 1}} \vee \underbrace{M_S^{[2]}}_{\text{length 2}} \vee M_S^{[3]} \vee M_S^{[4]} \vee M_S^{[5]} \leftarrow \text{path of length 5}$$

or

M_{S^+} using Warshall's Algorithm.

3b $(a,b) \in S^+$ if a goes to b with a non-zero length path

$$M_S = \begin{bmatrix} 0&1&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&1&0&0&0 \end{bmatrix} \qquad W_2 = \begin{bmatrix} 0&1&0&1&0 \\ 0&0&0&1&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&1&0&1&0 \end{bmatrix} \qquad W_4 = \begin{bmatrix} 0&1&0&1&1 \\ 0&0&0&1&1 \\ 0&0&0&1&1 \\ 0&0&0&0&1 \\ 0&1&0&1&1 \end{bmatrix}$$

$W_1 = M_S$

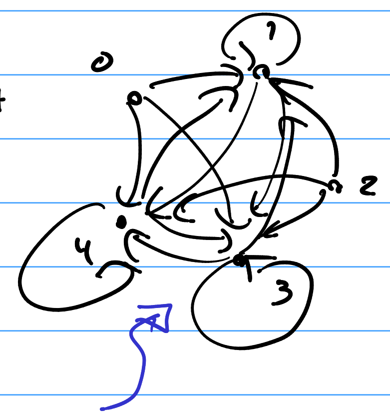
0, 1, 3, 4

$W_2 = W_3$

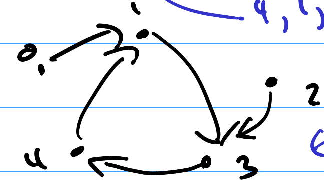
$$M_{S^+} = W_5 = \begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 0&1&0&1&1 \\ 1 & 0&1&0&1&1 \\ 2 & 0&1&0&1&1 \\ 3 & 0&1&0&1&1 \\ 4 & 0&1&0&1&1 \end{array} \longrightarrow S^+$$

$\boxed{a}$ path from 3 to 3

4, 1, 3



$$= \begin{bmatrix} 0&1&0&0&0 \\ 0&0&0&1&0 \\ 0&0&0&1&0 \\ 0&0&0&0&1 \\ 0&1&0&0&0 \end{bmatrix} \longleftrightarrow$$



Functions

relation such that every $a \in A$ goes to exactly one $b \in B$

$f : A \to B$

Notation

1. $f(a) = b$
2. A is domain a is pre-image of b
3. B is codomain b is image of a
4. range = set of all images

$$\text{range} = \{ e \mid \exists a \;\; f(a) = e \}$$

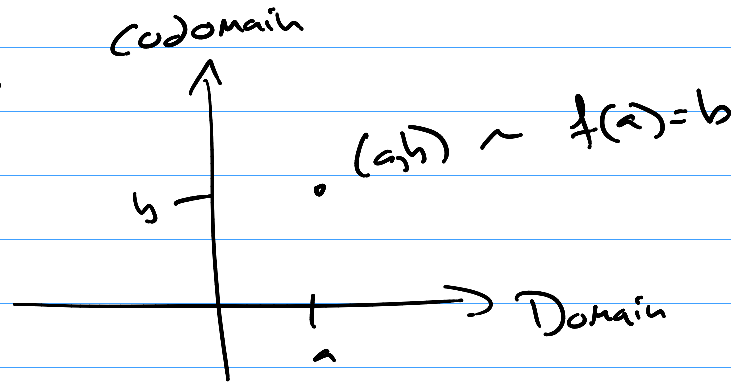
(2) functional notation of sets

is $S \subseteq A$

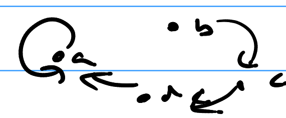
$$f(S) = \{ e \mid s \in S \;\; f(s) = e \}$$

So.. $f(A) = \text{range}$

Calculus / functions



(Ex) R on $\{a, b, c, d\}$



$$R = \{ (a,a), (b,c), (c,d), (d,a) \}$$

ref? no, b/c $(b,b) \notin R$ $(c,c) \notin R$ $(d,d) \notin R$

irref? no, b/c $(a,a) \in R$

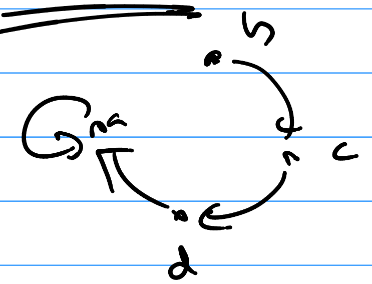
sym? no, b/c $(b,c) \in R$ but $(c,b) \notin R$

antisym? yes

asym? no b/c not irref.

trans? no b/c $(b,c) \in R \land (c,d) \in R$ but $(b,d) \notin R$

function? $R = \{(a,a), (b,c), (c,d), (d,a)\}$



Domain = $\{a, b, c, d\}$

Yes. b/c every $e \in$ Domain goes to exactly one $e \in$ Codomain.

Range = $\{a, c, d\}$

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Sub-classes of functions -

① one-to-one or injective function

$$\forall a \forall b \, (a \neq b \rightarrow f(a) \neq f(b))$$

$$\equiv \forall a \forall b \, (f(a) = f(b) \rightarrow a = b)$$

Ex $R = \{(a,a), (b,c), (c,d), (d,a)\}$

is **not** an injection b/c $a \rightarrow a$ and $d \rightarrow a$

② onto or surjection

(or Range = Codomain)

$\forall b \, \exists a \quad f(a) = b$

all elements in codomain

(3) bijection is one-to-one and onto

So far composition of functions $f \circ g$

we can define inverse functions.

$$(f \circ f^{-1})(x) = x$$

$$(f^{-1} \circ f)(x) = x$$

f^{-1} only exists if f is a bijection.