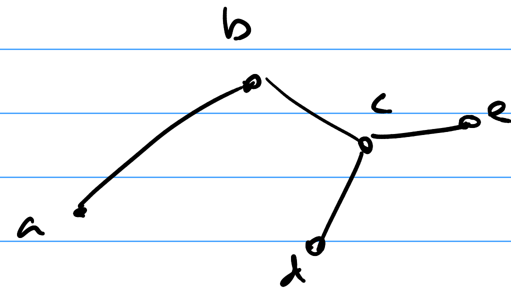
Math 322

Q's Due Wed 9.2(3)
9.4(2, 4, 5, 6, 10)

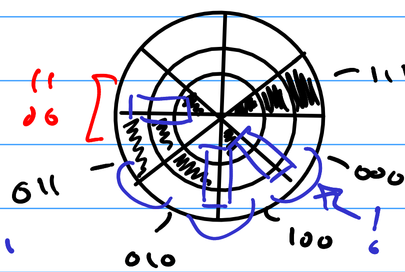
Applications → Path? Meaning?

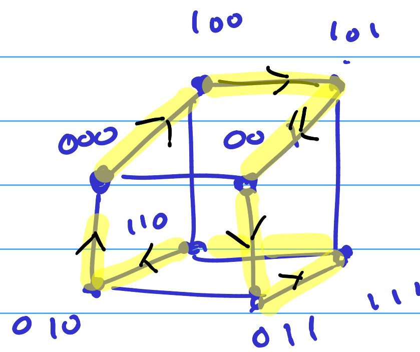


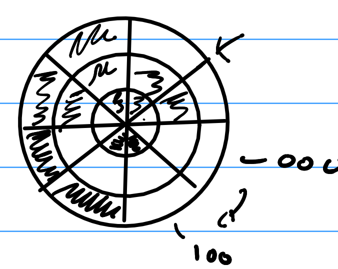
Ex handle 0, 1, 2, 3, 4, 5, 6, 7

0 : 000
1 : 001
2 : 010
3 : 011
4 : 100
5 : 101
6 : 110
7 : 111

______ 0 or 1
______ 0 or 1
______ 0 or 1





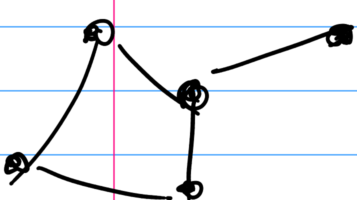


Paths in Applications

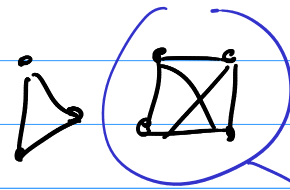
① Connectivity.

Undirected Graphs

G is connected if there is a path between any two vertices



G
(connected)



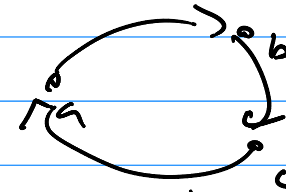
G
(not connected)

connected component

Directed Graphs

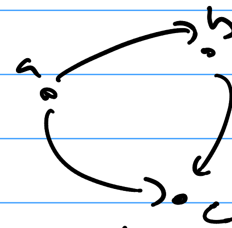
① Strongly Connected

G is strongly connected if there are paths to/from any two vertices.



is strongly connected.

Note



is not strongly connected.

② G is called weakly connected if we would ignore direction and see the underlying undirec. graph is connected.

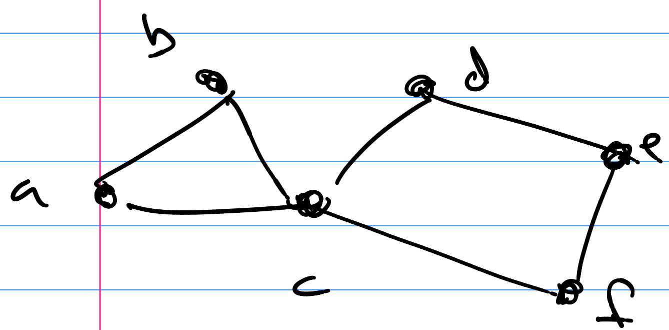
We can use $A_G \vee A_G^{[2]} \vee \cdots \vee A_G^{[|A|]}$ to see if it is connected / strongly connected.
b/c all 0's mean no path of any length between those vertices.

Now: if we have a connected graph .. how **strong**?

i.e. does removal of v_i or e_i make the graph disconnect?

① Vertex Cut $\equiv$ set of vertices whose removal will disconnect the graph.

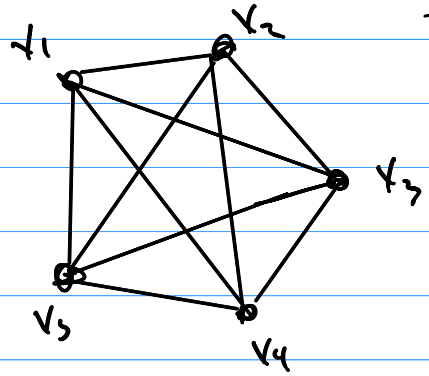
- $K(G)$ = min sized vertex cut.
- if $K(G) = 1$ ← remove 1 "special" vertex
  → call that vertex a $\boxed{\text{cut vertex}}$



Vertex cuts $\{d, f\}$
$\{c, d, e\}$
$\{c\}$ ← c is a cut vertex

$K(G) = 1$

K_5

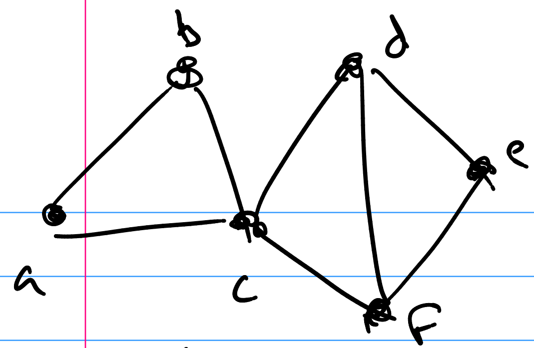


we can't actually disconnect K_n using vertices! The best I can do is "isolate" by removing everyone else

$K(K_n) = n-1$

② **Edges** → how many edges to disconnect graph?

Set of edges that disconnect is $\boxed{\text{edge cut}}$



G

<u>ex's</u> $\{\{a,c\},\{c,f\},\{c,d\}\}$

$\{\{a,b\},\{a,c\}\}$

$\lambda(G)$ = min edges to disconnect

$\lambda(G) = 2$

thm $1 \le \kappa(G) \le \lambda(G) \le \min_{v \in V} \deg(v) \le n-1$

gud K_n is the most connected graph

$\kappa(K_n) = \lambda(K_n) = n-1$