Math 322

Q's G is a tree if it has no simple circuits

logically equiv

1. G is a tree
2. uniq simple path between distinct vertices
3. if G is connected and $G = (V, E)$
   $(V, E - \{e\})$ is disconnected
4. if G is connected and has no cycles
   $(V, E \cup \{e\})$ creates a cycle
5. G is connected and $|E| = n - 1 = |V| - 1$

Apps

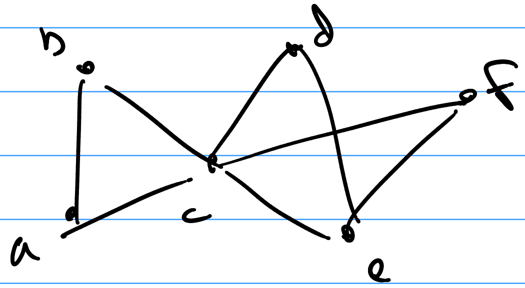
1. remove edges from $G = (V, E)$ until it becomes a tree

Spanning Tree $G' = (V, E')$

$\Leftarrow$

$E' \subseteq E$

$G =$



$|E| = 8$ $\rightsquigarrow$ $|E'| = 5$

$|V| = 6$

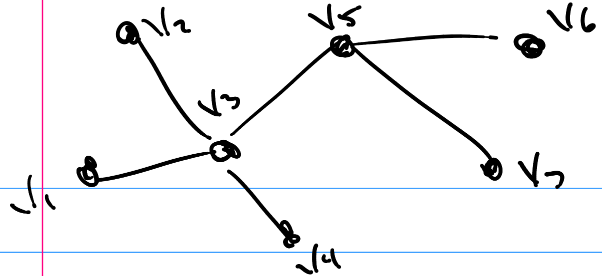
? G' is a tree ?

Note: we can modify trees by adding direction.

Def: rooted tree : tree where we

1. choose a root
2. directions all point away from the root for all paths

tree

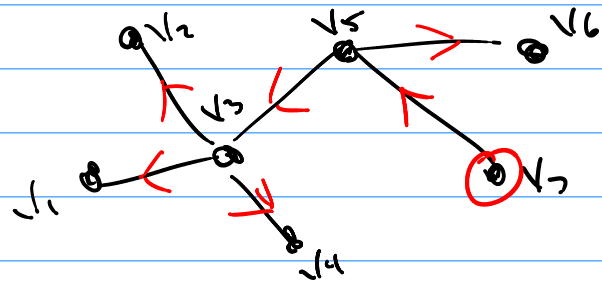


G is a tree ← connected

$|V| = 7$

$|E| = 6$

rooted tree

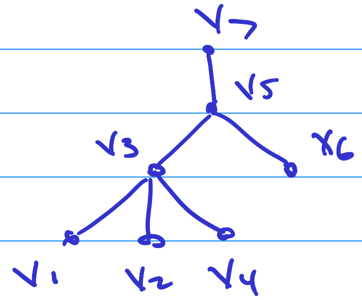


① pick V_7 as root

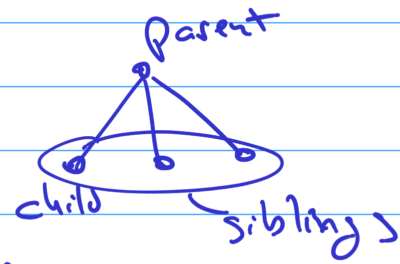
② Fact: unique simple path between any distinct vertices

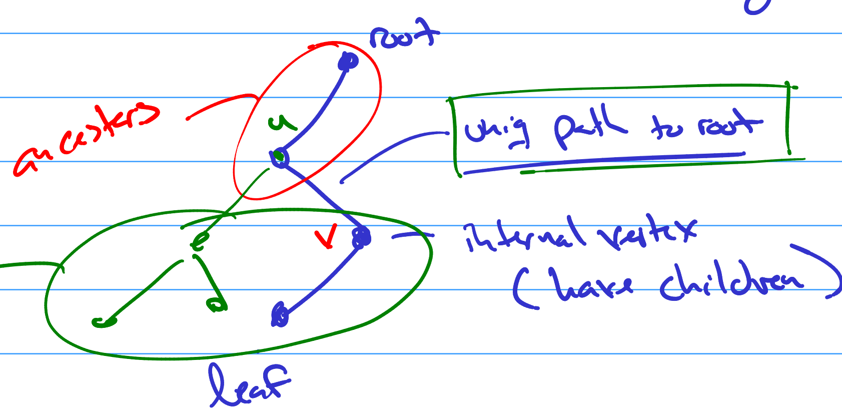
⇒ we have uniq simple paths from V_7 to each other V_i.

③ redraw (direction is down)



terms:

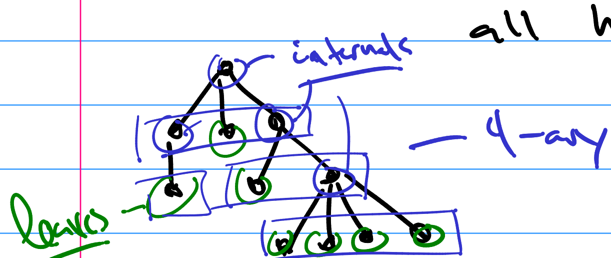


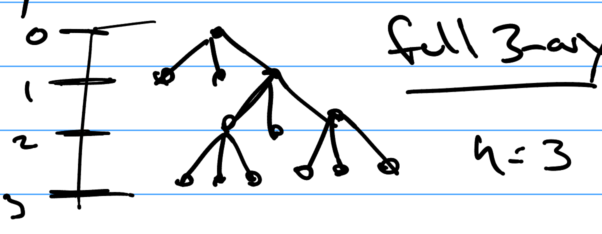


Terms of rooted trees

① m-ary tree: in the tree you see internal vertices have at most m-children

② full m-ary tree: in the tree internal vertices all have exactly m-children





(3) height = longest path back to root

(4) Facts: $n = |V|$ $\quad |E| = n-1$ (it's a tree!)

(1) $l \equiv$ leaves $\quad i \equiv$ internal vertices

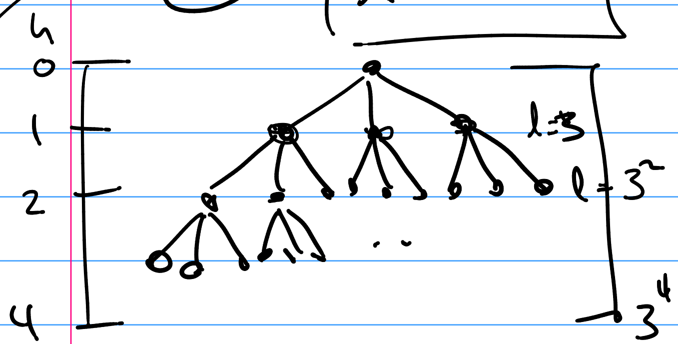
$$\boxed{n = i + l}$$

(2) $|\text{all children}| = n-1$

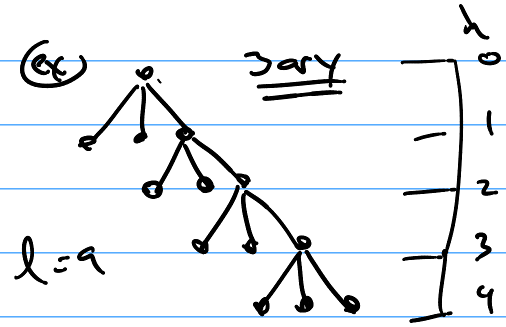
(3) full m-ary $\quad |\text{all children}| = mi$

(4) full m-ary tree $\quad \boxed{n = mi + 1}$

(5) $\boxed{l \le m^h}$



(ex) 3ary



(6) $l \le m^h \rightsquigarrow \log_m l \le \log_m m^h$

$$\log_m l \le h$$

so $\boxed{\lceil \log_m l \rceil \le h}$

Minimal Spanning Trees

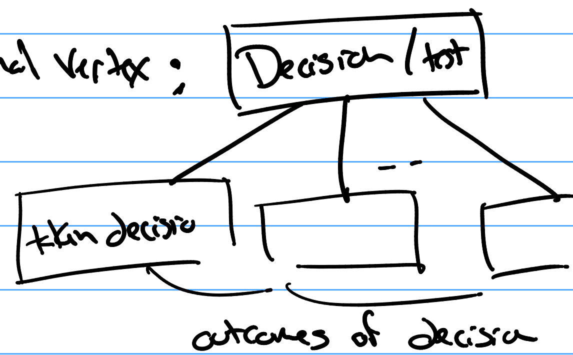
① Spanning Tree
② $\sum_{E'} w(e)$ is minimal.

What to do with a tree?

① Decision Trees

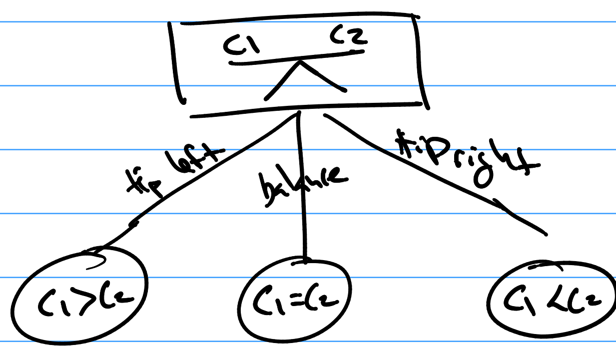
a) internal vertex:

b) children



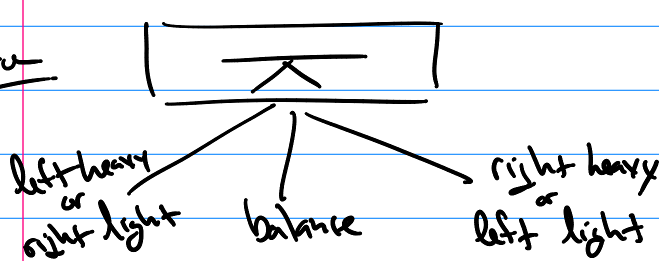
c) leaf: end result

Ex weight two coins on a balance scale



Ex Given 4 coins 3 are good 1 maybe light or heavy.

decision



Outcomes

1 heavy, 1 light
2 H, 2 L
3 H, 3 L
4 H, 4 L or all ok

So we have a 3-way tree with $l = 9$

$\rightarrow$ $h \geq \lceil \log_3 9 \rceil = 2$ $\leftarrow$ best case

c_1, c_2
c_3, c_4

Make a tree