Math 322

Due Next Wed 11.1 (1, 2)
11.2 (3)

Algebraic Systems: [V ; list of binary / unary operators]
$\underbrace{V}_{\text{domain}}$

→ list of possible properties for operations on V
ex: closure, associative, commutative, distributive, etc..

Abstraction

less → More

concrete → **Axiomatic** → **Universal**

- ex propositional logic
- ex elementary algebra
- ex Graph theory
- ex number theory &
- ex Set theory
- ex linear Algebra
- ex The Calculus

Axiomatic:
- basic elements of a concrete example
- basic properties of a concrete example (axioms)

→ construct the concrete example is created from these

Universal study any concepts universal to any (algebra) math system

ex: **isomorphism**, **subsystem**, . . .

Graph theory

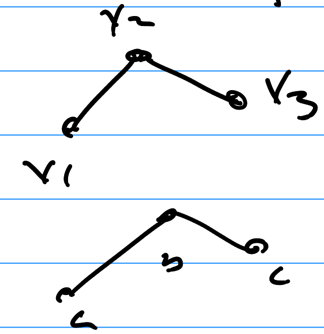
A_{G_1} ⇝ Isomorphism (bijection from $G_1 \to G_2$)

such that A_{G_1} same A_{G_2} under the bijection

called similar

$$B = E^{-1} A_G E$$

row swap col swap



---

ex

Monoid

def: $M = [V; *]$ — domain, binary operator

and ① there is an identity element in V, e
$(a * e = e * a = a)$
② associative property

ex V is any bit string and λ is the empty string

$B = \{0, 1\}$ any string of any length B^*

operator is concatination $01 \frown 0011 = 010011$

① $b \frown \lambda = b$ $\lambda \frown b = b$

so λ is an identity for concat.

② $(01 \frown 11) \frown 101$ vs $01 \frown (11 \frown 101)$

$(0111) \frown 101$ $01 \frown (11101)$

0111101 0111101

looks like we want to show $(b_1 \frown b_2) \frown b_3 = b_1 \frown (b_2 \frown b_3)$

associativity.

so $\boxed{[B^*, \frown] \text{ is a Monoid}}$ ← Concrete example of a Monoid.

Thm (something I can prove)

$[V, *]$ is a monoid and we have commutativity for $*$ $(a*b = b*a)$

then $(a*b)*(a*b) \overset{assoc}{=} a*(b*a)*b$
$= a*(a*b)*b$
$= (a*a)*(b*b)$

---

ex $(2 \cdot x)^2 = (2 \cdot x) \cdot (2 \cdot x) = (2 \cdot 2) \cdot (x \cdot x) = 2^2 \cdot x^2$

Def Group

$[G; *]$ — domain (G), binary operator ($*$)

1. $*$ is associative
2. identity element $e*a = a*e = a$
3. for $a \in G$ there is a b such that $a*b = b*a = e$ (inverse)

Def: Abelian Group

$[G; *]$ is a group and
$a*c = c*a$ (commutative)

Axiomatic Method starting with the facts of

$[G; *]$ is a group ...

→ what can I prove?

something in G

something in G

Why? consider: $a * (?) = b$

something I don't know

$2 * ? = 6$

$$\boxed{\begin{aligned} a * X &= b \\ a^{-1} a * X &= a^{-1} * b \\ e * X &= a^{-1} * b \\ X &= a^{-1} * b \end{aligned}}$$

11.3 General Properties of Groups

facts of Groups that I can prove.

Thm: Identities of G are unique.

in the elements of G there is only one identity element.

pf: assume e, f are both identities and $e \neq f$.

so $e = e * f = f$ so $e = f$. $= f$