

Math 322

Exam 1 11 probs @ 10pts / 100pts = 100%

Relations: probs 1-7

Functions: 10, 11

① given $R = \{ (a,b) \mid \text{predicate}(a,b) \}$ (Set builder)
or (list)

(ex) $R = \{ (x,y) \mid x \leq y+2, x \in \mathbb{R}, y \in \mathbb{R} \}$

(ex) $R = \{ (c,d) \mid \underline{\text{chair } c} \text{ is beside } \underline{\text{chair } d} \}$

→ state all properties (with logic)

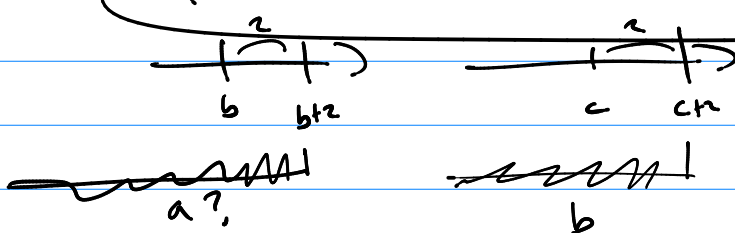
→ does R have a specific property (for each)

Reflexive: $\forall c (c R c) \equiv \text{for all reals } x, x \leq x+2$ Yes

Irreflexive: $\forall c (c R c) \equiv \text{for all reals } x, x > x+2$ No

→ counter example
 $0 \not> 0+2$

Transitive $\forall a \forall b \forall c (a R b \wedge b R c \rightarrow a R c)$
 $\equiv (a \leq b+2 \wedge b \leq c+2) \rightarrow (a \leq c+2)$



$$b = 3 \quad 2$$

$$\text{so } a \leq 5$$

$$a = 5, b = 3, c = 1$$

$$c = 1 \quad 2$$

$$b \leq 3$$

$$5 \leq 3+2 \quad \wedge \quad 3 \leq 1+2 \quad \text{but} \quad 5 \not\leq 1+2$$

$$a \quad b \quad b \quad c \quad a \quad c$$

No $\rightarrow$ counterexample

Q R on A

$R: A \rightarrow A$

but domain, codomain on A .

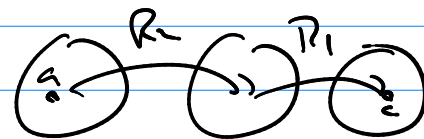
R on $\{.., -2, -1, 0, 1, 2, ..\}$
all ints.

(vs) R on $\{2, 3, 5, 7, 11, ..\}$
primes.

but use: $R = \{(a, b) \mid a + b = 0\}$

irreflexive: $\forall e (e \notin R)$
 $\equiv$ "no e is $e + e = 0$ "
False $\forall e \quad \underline{0 + 0 = 0}$

irref. $\forall e (e \notin R)$
 $\equiv$ "no e is $e + e = 0$ "
true.



(2) given a few relations $R_1, R_2, ..$

can you state $R_1 \cup R_2$ or $R_1 \cap R_2$ or $R_1 \circ R_2$
(any rep. you want. - list
- set builder
- digraph
.. etc)

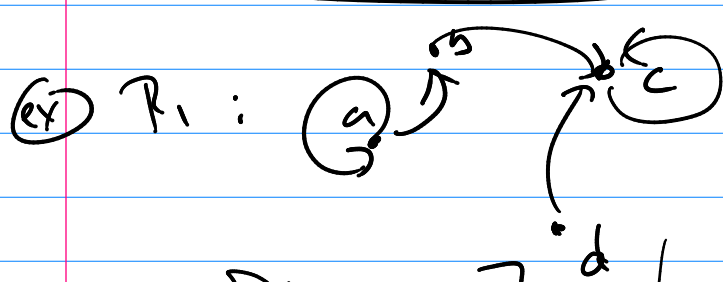
(I may state what I want)



(given one
+ you give other two)

④ $R_1 \cup R_2$? $R_1 \cap R_2$? $R_1 \circ R_2$? $R_2 \circ R_1$?

using Matrices



$$M_{R_1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

R_2 : $\{(a,b), (a,d), (b,b), (b,c), (c,d), (d,d)\}$

$$M_{R_2} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$M_{R_1 \cup R_2} = M_{R_1} \vee M_{R_2} = ? \quad M_{R_1 \cap R_2} = M_{R_1} \wedge M_{R_2} = ?$$

$$M_{R_1 \circ R_2} = M_{R_2} \odot M_{R_1} = ?$$

⑤/⑥ closures!

- 5) a) ref. closure
- 5) syn. closure

- 6) trans closure
- a) $M_{R^+} = M_R \vee M_R^{(2)} \vee \dots \vee M_R^{(|A|^3)}$
- b) Warshall's

7) (2) Equiv. Relations / classes.

→ 7) show R is ref, sym, trans.

e) equiv. class for a specific element.

$$[e]_{\sim_{\text{equiv. relation}}} = \{s \mid eRs\}$$

1) a) Poset + b) Hasse Diagram

↓
show R is ... ref, anti sym, trans.

10) for R on domain, codomain

a) give a non-function R

b) give a function that is ...

is onto	is one-to-one
is onto	is not one-to-one
is not onto	is one-to-one
is not onto	is not one-to-one

be able
to do
all 4,

(ex) R on $(\mathbb{R})$ to $(\mathbb{Q})$

R is not a function example.

a) set builder $R = \{(a,b) \mid b = \pm |a|\}$

or (ex) $a=b$ but $\pi \notin \mathbb{Q}$ so $\pi \rightarrow ?$ nowhere!
or $\pi \rightarrow +|\pi|=4$
 $\pi \rightarrow -|\pi|=-4$

is onto and not 1-1

$$\sqrt{2} \in \mathbb{R}$$

$$\sqrt{2} \notin \mathbb{Q}$$

a map to closest $b \in \mathbb{Q}$

$$\sqrt{2} \rightarrow$$

$$\text{where } |b - a| < 10^{-6}$$



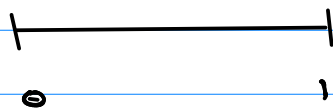
need to study this

(11)

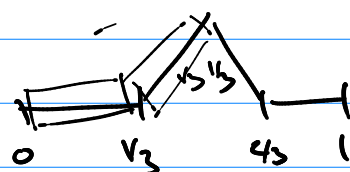
bijection question.

→ show $|A| = |B|$

find bijection from A to B.



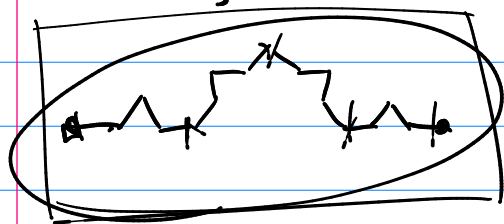
$$l = 1 = \left(\frac{4}{3}\right)^0$$



$$l = \left(\frac{4}{3}\right)^1$$

$$1D = \infty$$

$$2D = 0$$



$$l = \left(\frac{4}{3}\right)^2$$

:

$$l \rightarrow \infty$$

$$2 = \frac{l_4(1)}{l_4(3)}$$

1D

$$(l)^1 = -$$

2D

$$(l)^2 = B$$

3D

$$(l)^3 = B^3$$

$$(l)^{(2)}$$