Math 322

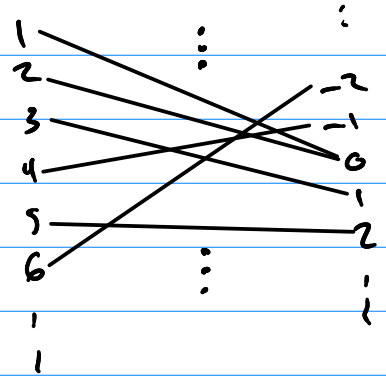
Q's Exam #/40 = ?

Function: onto is Codomain = Range

all $b \in$ Codomain have $\geq$ preimage

$A(a) = b$

(ex) Domain Codomain



not 1-1, is onto

$|A| = |B|$ if we have a bijection

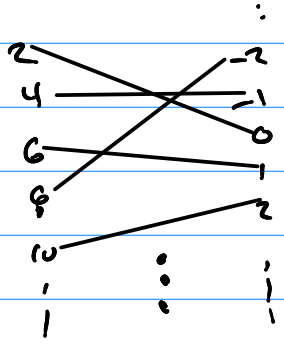
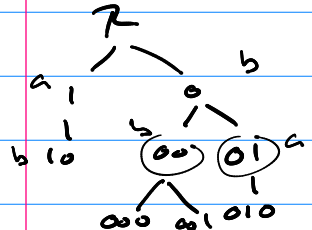
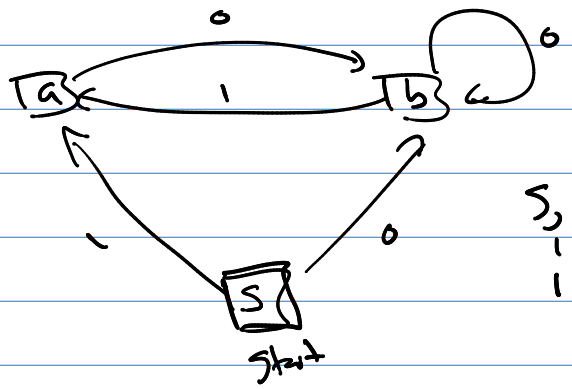


Figure 9.1.4



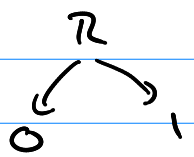


Path = string.

S, a, b, a, b

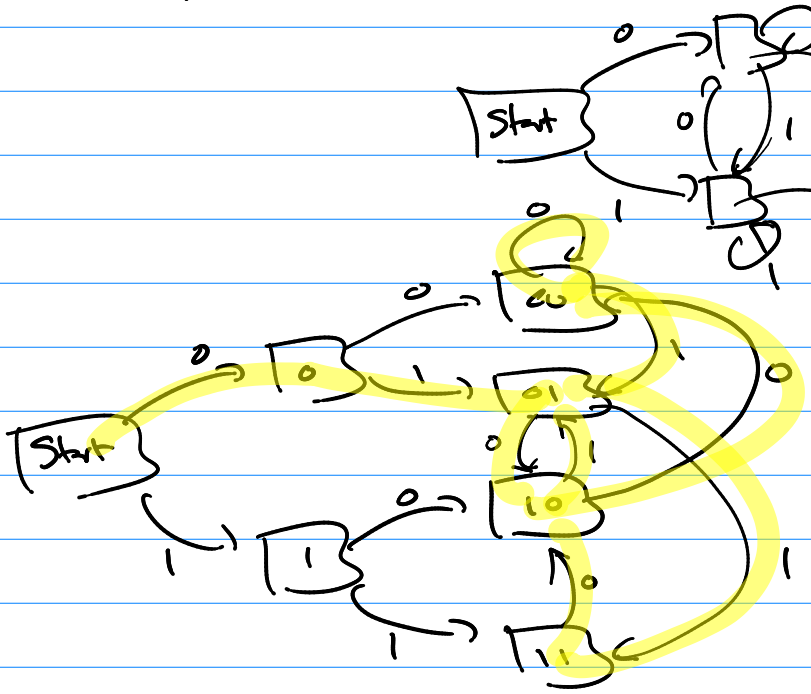
1 1 0 1 0

Set of strings of 0's and 1's



R, 0, 1, 00, 01, 10, 11

empty string, 000, 001, 010, 011, 100, 101, 110, 111



0110100 01

New graphs from old graphs

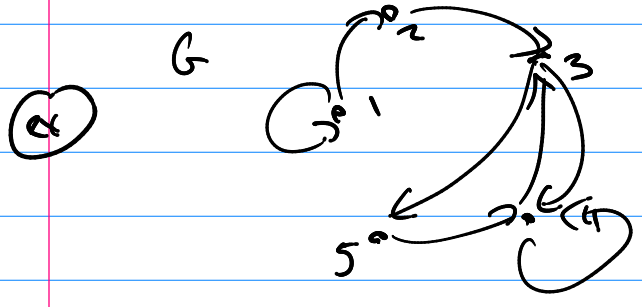
(1) Union $G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$

(2) Subgraphs H subgraph of G

if $H = (V_1, E_1)$ $G = (V_2, E_2)$

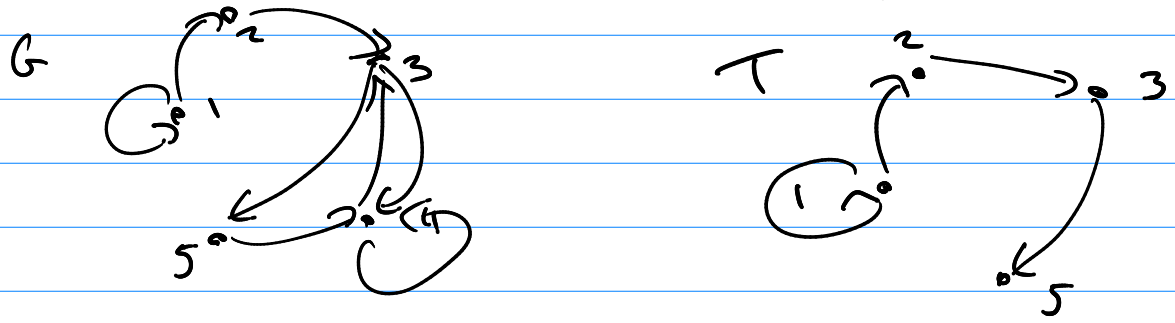
then $V_1 \subseteq V_2$, $E_1 \subseteq E_2$

(a)



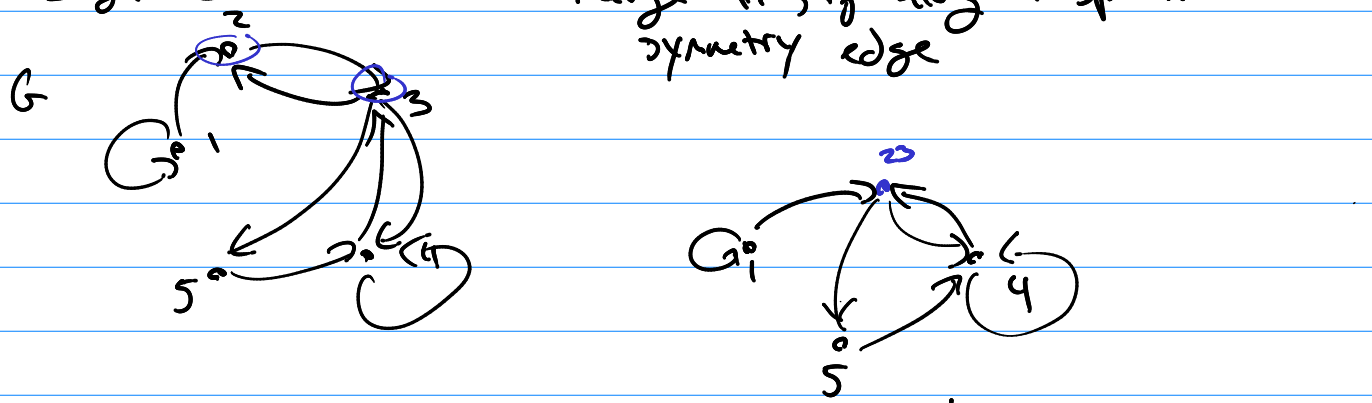
③ Induced Subgraph take $V_1 \subseteq V_2$

and all edges involving $v \in V_1$

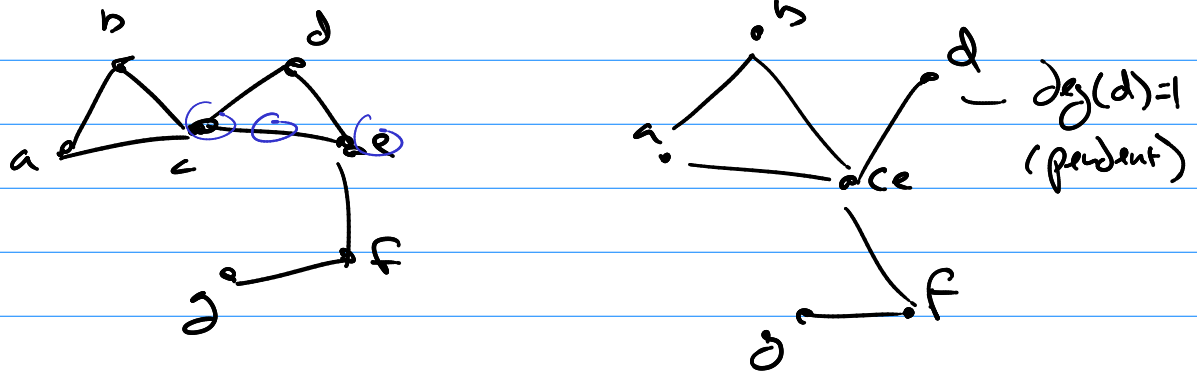


④ Edge Contraction

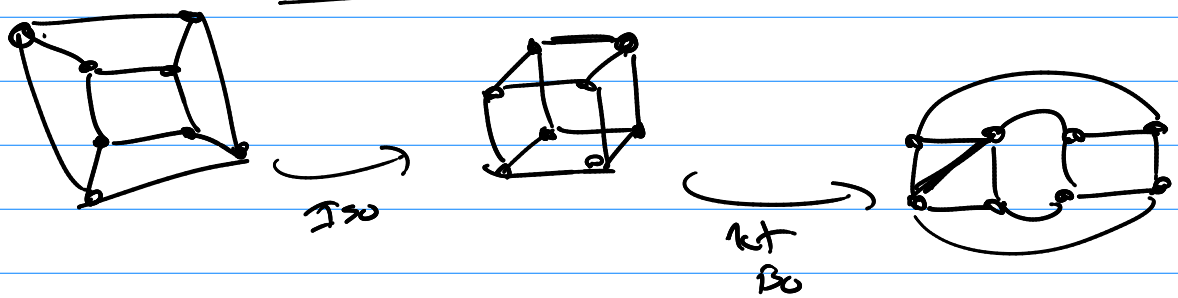
Merge V_i, V_j along a specific symmetry edge



Ex



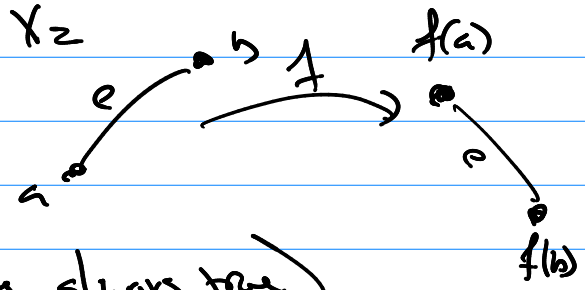
"Same" = Isomorphic Graphs



G_1 to G_2 Are they isomorphic?

(V_1, E_1) (V_2, E_2)

required: bijection from V_1 to V_2 that preserve edges.



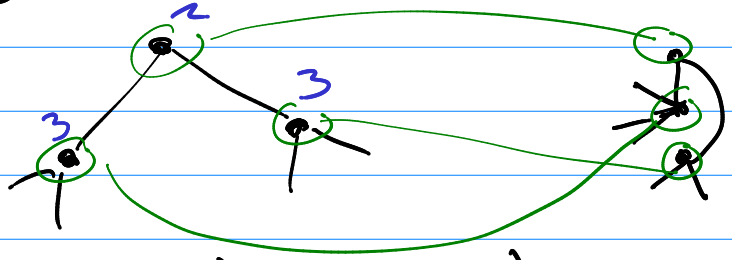
Invariants (when a bijection there are always true)

① $|V_1| = |V_2|$, $|E_1| = |E_2|$

② degree of vertices do not change

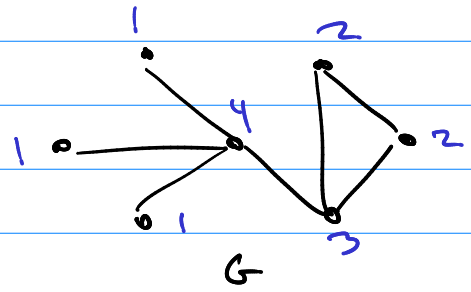
$$\deg(v) = \deg(f(v))$$

③ degree of neighbors do not change

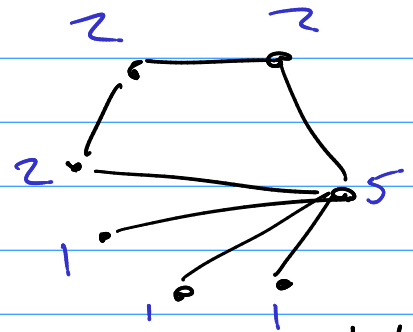


④ paths must be preserved.

Ex



G



H

$|V| = 7$
$|E| = 7$

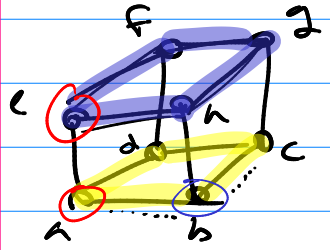
don't have same deg's

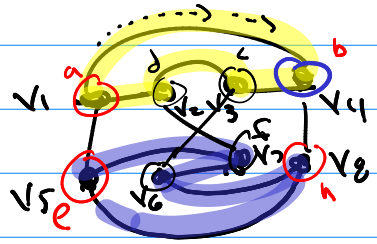
$|V| = 7$
$|E| = 7$

→ Not Isomorphic

to show G_1 and G_2 are isomorphic we need to show there is a (bijection) that preserves edges.

find one





A

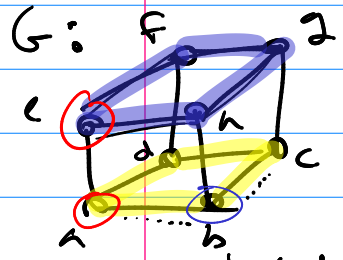
a → v_1
b → v_4
c → v_3
d → v_2
e → v_5
f → v_7
g → v_6
h → v_8

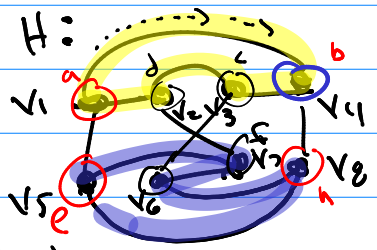
isomorphism? (show)

use matrices to show isomorphic

Adjacency Matrix $A_G = [a_{ij}]$ a_{ij} = # of edges from v_i to v_j

if $A_{G_1} = A_{G_2}$ in the order of the isomorphism call them isomorphic





$A_G =$

	a	b	c	d	e	f	g	h
a	0	1	0	1	1	0	0	0
b	1	0	1	0	0	0	0	1
c	0	1	0	1	0	0	1	0
d	1	0	1	0	1	0	0	0
e	1	0	0	0	1	0	1	0
f	0	0	0	1	0	1	0	1
g	0	0	1	0	0	1	0	1
h	0	1	0	0	1	0	1	0

$A_H =$

	v_1	v_4	v_3	v_2	v_5	v_7	v_6	v_8
v_1	0	1	0	1	1	0	0	0
v_4	1	0	1	0	0	0	0	1
v_3								
v_2								
v_5								
v_7								
v_6								
v_8								

A

a → v_1
b → v_4
c → v_3
d → v_2
e → v_5
f → v_7
g → v_6
h → v_8