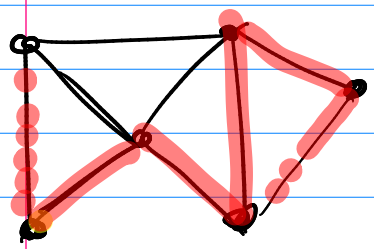
Math 322

Applications

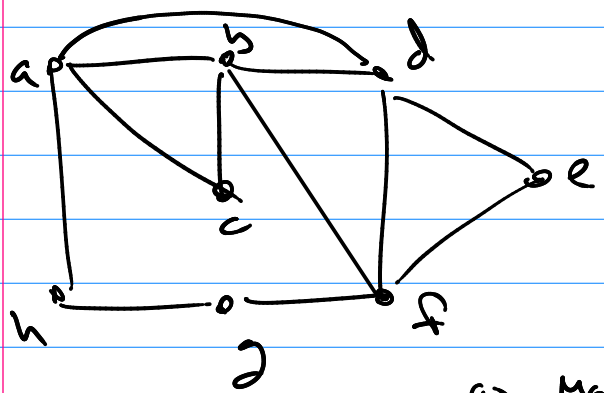
① Eder / Hamilton Circuits, Paths

Def: Eder Circuit: Simple circuit that includes all edges of $G=(V,E)$



Th^m You have an Eder Circuit iff $\deg(v)$ is even for all $v \in V$.

How to create an eder circuit you just add "cycles" into "cycles" "loops" into "loops"

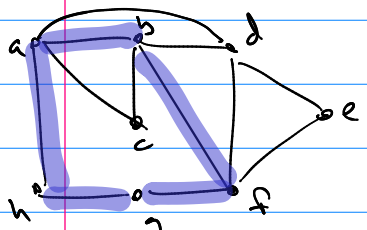


all $\deg(v)$ are even so Eder circuit exists!

Make it

① Make any simple circuit

a, h, g, f, b, a

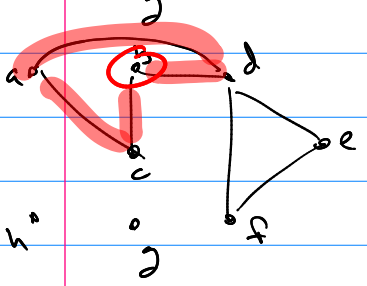


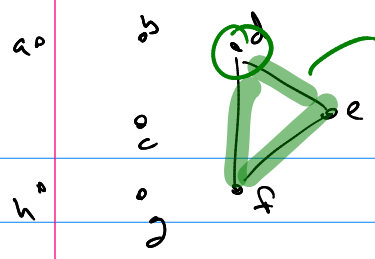
② add loop into our circuit

a, h, g, f, b, a

b, d, a, c, b

a, h, g, f, b, d, a, c, b, a

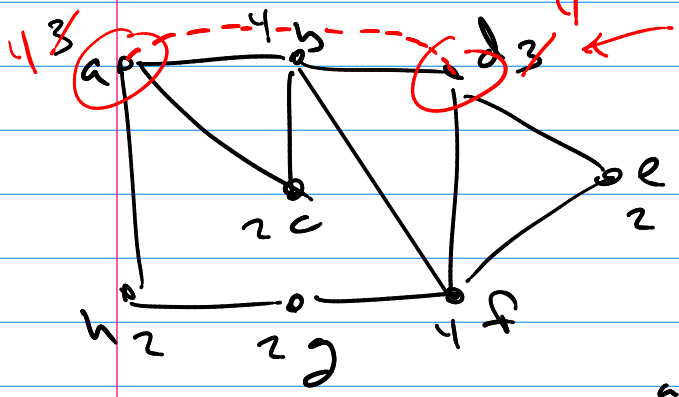




a, h, g, f, b, d, e, f, d, a, c, b, a

Euler Path (non-circuit)

Th² G has an euler path (non-circuit) iff $\deg(v)$ is even for all vertices **except** exactly two are odd. And the path starts/ends at the odds.

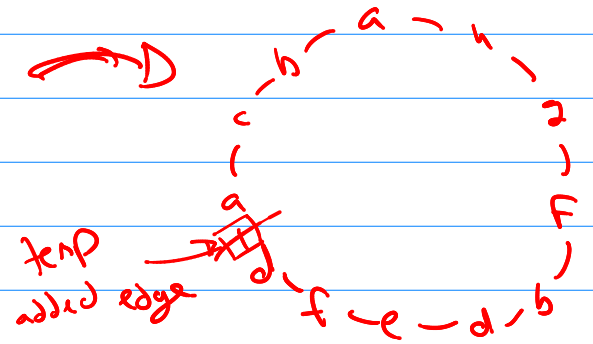


no euler circuit b/c a, d

but If I add a temp edge {a, d}

this graph does have an euler circuit

a, h, g, f, b, d, e, f, d, a, c, b, a

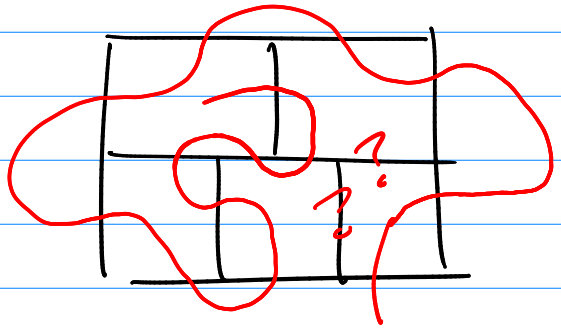


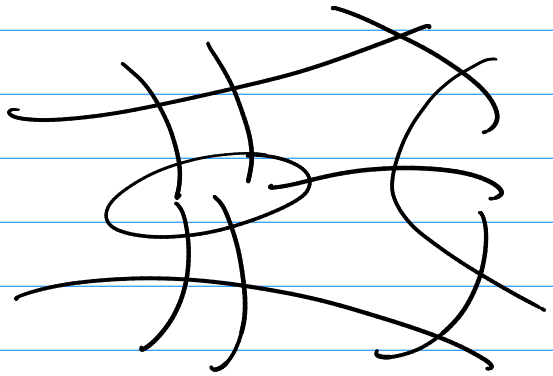
Cut our temp edge, and we get our euler path

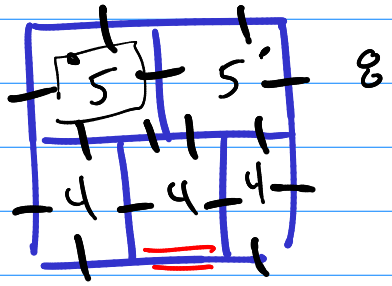
a, c, b, a, h, g, f, b, d, e, f, d

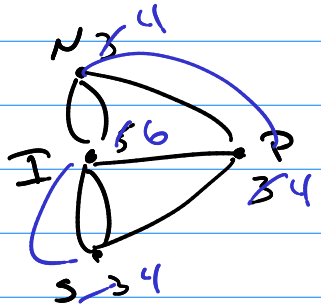
Apps & Euler Circuits Paths

1. tracing
2. routes that don't reuse edges (delivery)
3. cut puzzles

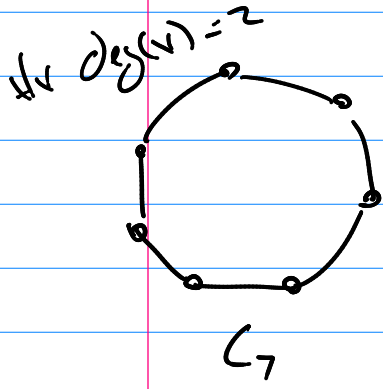






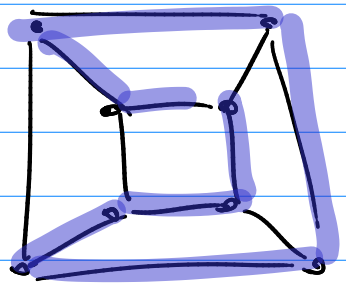


Hamilton Circuit: Simple circuit that visits each vertex exactly once, except 1st vertex = last vertex.

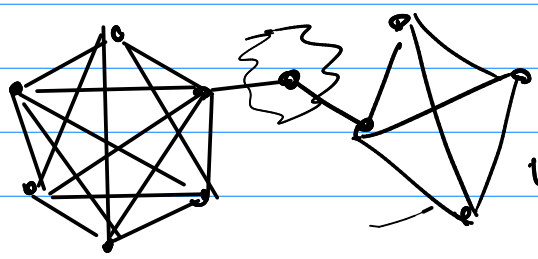


has an Hamilton Circuit

C_7



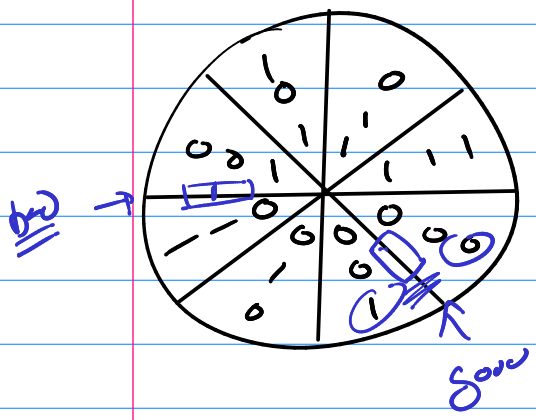
Q_3

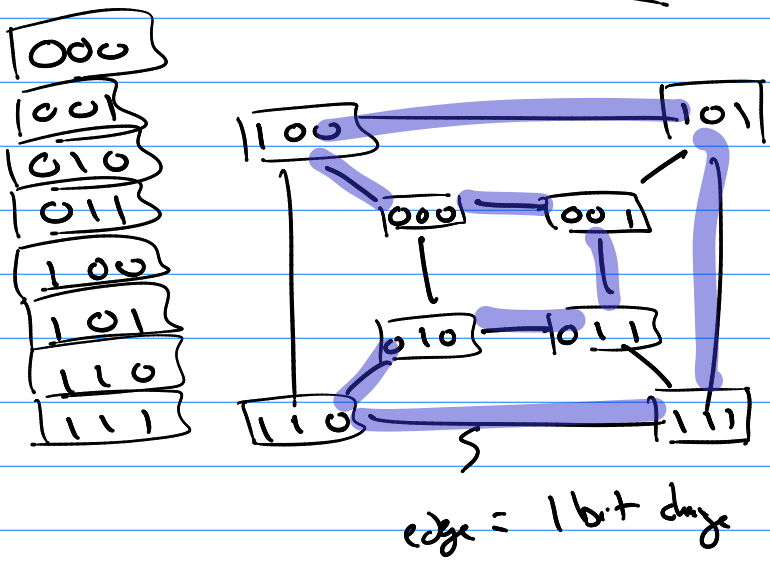


no Hamilton circuit

all graphs with hamilton circuits

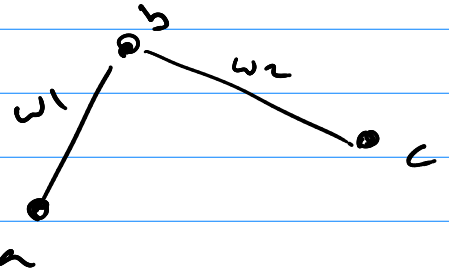
th^n if $\frac{\deg(x)}{} \geq \frac{|V|}{2}$ then G has a hamilton circuit.





② Weighted Graphs and Minimal paths

$G = (V, E)$ add $w(e)$ = weight of edge e



$w(\{a,b\}) = w_1$

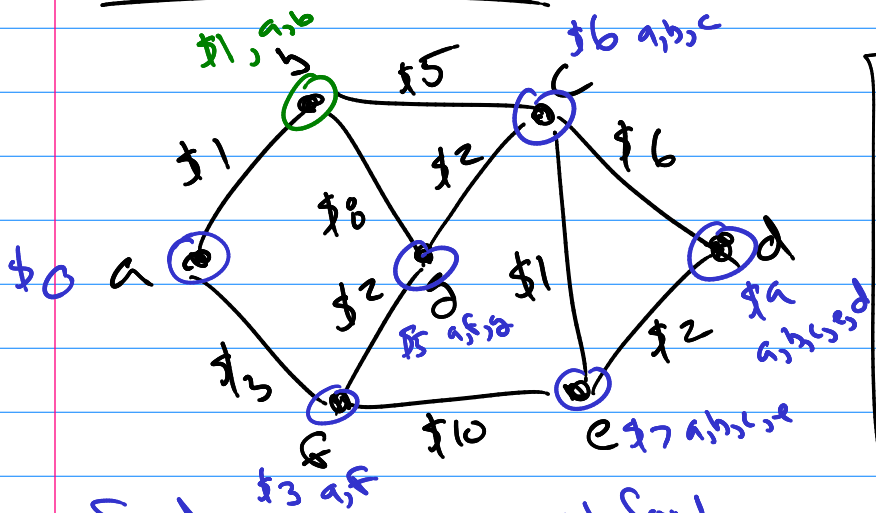
$w(\{b,c\}) = w_2$

a) length of a path is the sum of the weights.

b) we can restate old length by using $w(e) = 1$.

1st Minimal Paths Find all minimal paths from a fixed vertex to all the others..

Dijkstra's Algorithm



make a list of a to (everyone else)

a to a	
a to b	
a to c	
a to d	path and cost for cheapest way to do it.
a to e	
a to f	
a to g	

Scratch

Found

a to a

b $1 a,b

f $3 a,f

g $5 a,f,g

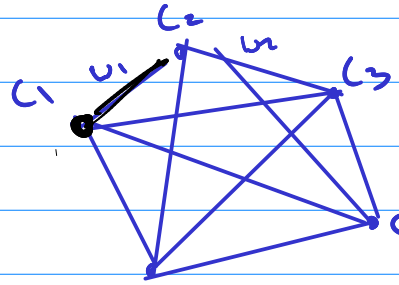
not found

c, d, e

Traveling Salesman problem → find the minimum Hamilton circuit

consider K_n as a weighted graph $|V| = n$

$|E| = \frac{n(n-1)}{2}$



~ City #4

b/c $\deg(v) = n-1 \geq n/2$ always for K_n we have hamilton circuits

now I have $|\text{hamilton circuits}| = (n-1)(n-2)\cdots(2)(1) = (n-1)!$

to Solve traveling Salesman--

① Find [full] $(n-1)!$ circuits

② Find the min